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CONFIDENCE AND TOLERANCE BOUNDS AND A NEW GOODNESS-OF-FIT TEST FOR TWO-PARAMETER WEIBULL OR EXTREME-VALUE DISTRIBUTIONS (WITH TABLES FOR CENSORED SAMPLES OF SIZE 3(1)25)

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AIR FORCE SYSTEMS COMMAND
United States Air Force

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FOREWORD

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ABSTRACT

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The second paper, by Nancy R. Mann and Kenneth W. Fertig, gives tables of values for obtaining confidence and tolerance bounds from best linear invariant estimates of parameters of extreme-value distributions. The tables apply to censored samples of size n, n = 3(1)25.

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A. A NEW GOODNESS-OF-FIT TEST FOR THE TWO-PARAMETER WEIBULL OR EXTREME-VALUE DISTRIBUTION WITH UNKNOWN PARAMETERS

SUMMARY

A new test of fit to the two-parameter Weibull or extreme-value distribution with unknown parameters is developed in this paper. This test is shown to have desirable power properties, relative to analogues of certain "classical" tests, against two important classes of alternatives. The test statistic is easy to calculate and can be used for censored samples. Percentage points and certain expected values which are needed to implement the test are provided for samples of size 3(1)25.

1. INTRODUCTION

There is a wealth of literature concerning what are known as "goodness-of-fit" tests or "distance" tests that a sample was drawn from a distribution with specified parameters. There is, for example, the Kolmogorov-Smirnov test (see Darling [6]), the Cramer-von Mises test (see Darling [6]), the Kuiper test [13], and the Anderson-Darling variation on the Cramér-von Mises test ([1] and [2]).

Some of these are discussed by Cox and Lewis [5], and the power of certain of these tests against various alternatives has been investigated by Brunk [4]. Birnbaum and Pyke [3] have examined other properties of statistics used in some of the tests.

David and Johnson [7] have shown that if the distribution in question has only a location and/or a scale parameter, goodness-of-fit tests, such as those listed above, which depend upon the probability integral transformation are independent of the true parameter values when one substitutes invariant estimates for them. (By an invariant estimator we mean here one which is a function of a maximal invariant and such that the mean squared error is independent of a location parameter, or invariant under translations.)

Using this fact, Lilliefors has tabulated critical values of the Kolmogorov-Smirnov statistic for normal and exponential distributions with parameters estimated from the sample ([15] and [16], respectively).

Motivated by Lilliefors' efforts, we first set out to construct tables similar to his for the two-parameter Weibull distribution where the parameters are replaced by estimates. However, in correspondence with the present authors, Lilliefors stated that a number of (unpublished) Monte Carlo trials he has conducted indicate that certain other of the classical distance tests, with estimates substituted for parameters, appear to provide higher power against certain alternatives than does the Kolmogorov-Smirnov. Thus, we enlarged the original scope of our wo· .. to consider these other tests as well as the Kolmogorov-Smirnov. Additionally, we were interested in tests based on a possibly censored sample, not necessarily on a full sample.

During the course of our investigation, a manuscript by van Montfort [25] came to our attention. Expanding on an observation of his, we developed two new statistics, which we call L and S, respectively. We have used these new statistics to provide a basis for a new test of fit to the extreme-value or Weibull distribution with very simple calculations involved and with power against the alternatives considered greater than that of any of the

classical tests mentioned earlier. The power comparisons are given in Section 4; the L and S statistics are discussed in Section 3; some preliminaries are dealt with in Section 2; computational details are given in Section 5; the percentiles of S and examples illustrating the use of the test are given in Section 6.

2. PRELIMINARIES

The three-parameter Weibull distribution is given by

$$P[T \le t] = \begin{cases} 1 - \exp\left\{-[(t-\mu)/\delta]^{\beta}\right\}, & t \ge \mu \\ 0 & , t < \mu \end{cases} \tag{1}$$

where μ, the location parameter, is unrestricted in sign; δ, the scale parameter, and β, the shape parameter, are positive. If $\mu = 0$, the distribution (1) is called the two-parameter Weibull distribution. If μ is known then, of course, the distribution of $T-\mu$ is the two-parameter Weibull.

If $\mu = 0$ and one makes the transformation $X = \ln T$ of the two-parameter Weibull random variable T, the distribution of X is called the extreme-value distribution. It is given by

$$P[X \le x] = 1 - \exp\left\{-\exp[(x-u)/b]\right\} \tag{2}$$

where $u = \ln \delta$ and $b = (1/\beta) > 0$. Note that u and b are, respectively, location and scale parameters of the extreme-value distribution.

The two distributions defined in (1) and (2) are sometimes referred to as the third and first asymptotic distributions of smallest (extreme) values or the Fisher-Tippett Type III and Type I distributions, respectively, (see Fisher and Tippett [9]). They are often used in survival analysis, both in the biological and in the physical sciences, as well as in many other applications (see, for example, Weibull [26]).

The tests of fit we discuss in this paper are based on the extreme-value distribution. To make a test of fit to the Weibull distribution, one first takes the natural logarithms of the supposed Weibull data.

Among the many procedures available for testing goodness of fit to a completely specified cumulative distribution function F_o, we considered the two-sided Kolmogorov-Smirnov, the Cramér-von Mises, the Anderson-Darling, and the Kuiper tests. Their definitions and computing forms are given below. All are based on uncensored ordered samples $X_1 < X_2 < \ldots < X_n$ and F_n, which denotes the empirical cumulative distribution function:

$$F_n(x) = (\text{number of } X_i\text{'s} \leq n)/n.$$

<u>The two-sided Kolmogorov-Smirnov statistic, D_n</u>

$$D_n = \sup_{-\infty<x<\infty} | F_n(x) - F_o(x) | = \max(D_n^+, D_n^-)$$

where

$$D_n^+ \equiv \sup_{-\infty<x<\infty} \left\{ F_n(x) - F_o(x) \right\}$$

$$= \max_{1\leq i\leq n} \left\{ i/n - F_o\ (X_i) \right\}$$

and

$$D_n^- \equiv \sup_{-\infty<x<\infty} \left\{ F_o(x) - F_n(x) \right\}$$

$$= \max_{1\leq i<n} \left\{ F_o(X_i) - (i-1)/n \right\};$$

<u>the Cramér-von Mises statistic, ω_n^2</u>

$$\omega_n^2 \equiv \int_{-\infty}^{\infty} \left\{ F_n(x) - F_o(x) \right\}^2 \ dF_o(x)$$

$$= 1/(12n^2) + (1/n) \sum_{i=1}^{n} \left[F_o(X_i) - (2i-1)/(2n) \right]^2 ;$$

<u>the Anderson-Darling statistic, W_n^2</u>

$$W_n^2 \equiv n \int_0^1 \left\{ \left[F_n(z) - z \right]^2 / \left[z(1-z) \right] \right\} \ dz$$

$$= -n - (1/n) \sum_{i=1}^{n} \left[(2i-1) \ln U_i + (2n-2i+1) \ln (1-U_i) \right],$$

where

$$U_i = F_o\ (X_i) ;$$

and <u>the Kuiper statistic,</u> V_n

$$V_n \equiv \text{range of difference between } F_o(x) \text{ and } F_n(x)$$

$$= D_n^+ + D_n^- .$$

We made Monte Carlo studies of the power of the analogues of these tests against two alternatives with, instead of a completely specified distribution function, invariant estimates substituted for the parameters appearing in $F_o(X_i)$ (where X_i is an ordered sample variate from the extreme-value distribution given by (2), or, equivalently, $T_i = \exp(X_i)$ is an ordered sample variate from the two-parameter Weibull distribution given by (1) with $\mu = 0$). Alternatives considered were that T_i was from a log-normal population (X_i was from a Gaussian population) and that T_i was from a particular three-parameter Weibull population, with $\mu = 10^6$ and $\beta = 1$ or $\beta = .5$.

The reason for using two-sided analogues of the classical tests is that for alternatives of interest, particularly those considered in the power tests, the function $F_o(X_i)$ under the alternative "crosses" $F_o(X_i)$ under the hypotheses at least once in the range of X.

The function $F_0(X_i)$ with Gaussian variates substituted should tend to be somewhat more symmetrically S-shaped than when the X_i are extreme-value variates. (There is, of course, some modification when estimates are substituted for parameters.) Hence, the two functions would be expected to cross in the range from minus to plus infinity. The three-parameter Weibull alternative would be used in cases in which one was testing that he had sampled from a two-parameter Weibull population with increasing failure rate ($\beta > 1$) and mode $\delta(1-1/\beta)^{1/\beta}$ near the minimum life μ of the alternative. The three-parameter Weibull alternative would have a constant or decreasing failure rate ($\beta \leq 1$) and the distribution would have no mode. In our power calculations, we have considered the special case $\mu/\delta = 10$, $\beta = 1$.

Comparison of the power of analogues of the classical tests with that of variations of the test based on the S statistic, mentioned in Section 1, for various sample sizes is given in Section 4. We also give, in Section 6, a table of 100 λth percentiles of the S statistic based on samples censored at the mth of n observations for λ = .75, .80, .85, .90, .95, .99; m = 3(1)n; n = 3(1)25.

3. THE L AND S STATISTICS

In [25], van Montfort states that for ordered variates $X_{i,n}$ from the extreme-value distribution, the quantities

$$\ell_i = (X_{i+1,n} - X_{i,n})/E\,(X_{i+1,n} - X_{i,n});\ i = 1, \ldots, n-1,$$

are approximately exponentially distributed with mean 1, variance close to 1, and they are nearly uncorrelated. (Following Tukey, van Montfort calls these ℓ_i's <u>leaps</u>; the numerators, $X_{i+1,n} - X_{i,n}$, are called <u>gaps</u>.)

Considering this observation of van Montfort, we noted that $2\ell_i$ is distributed approximately as chi-square with 2 degrees of freedom. Also, we noted that if the ℓ_i's were actually independent, rather than nearly uncorrelated, then for a sample censored at the <u>m</u>th of n observations and for $r + s \leq m \leq n$,

$$\frac{1}{r} \sum_{j=m-r}^{m-1} \ell_j \Bigg/ \left[\frac{1}{s} \sum_{j=1}^{s} \ell_j\right] \equiv L\,(r, s, m, n) \qquad (3)$$

would have approximately the Snedecor F distribution with 2r and 2s degrees of freedom. (We will henceforth suppress the arguments and write simply L.) In fact, for certain values of r, s, m, and n, we found that some percentiles of the L statistic agree with those of the appropriate F distribution

to within the limits of error of our Monte Carlo procedure. For other percentiles, the agreement was not quite so good. This means only that one cannot use already published tables to obtain all critical values for a goodness-of-fit test based on L. (We note that in any case, the statistic L is invariant under transformations of location and scale in X space.)

The important finding was that the power of a goodness-of-fit test based on L was better against the two alternatives considered than any of the four "classical" tests mentioned in Section 2.

We devised the test to exploit the fact that the right-hand tail of the extreme-value density function is "shorter" than that of usual appropriate alternative distributions, while the left-hand tail is "longer" (see Lieblein and Zelen [14], p. 291 for a graph of the density function of the extreme-value distribution). Thus, the "upper" gaps $X_{i+1,n} - X_{i,n}$ (upper meaning that i is closer to m-1 than to 1) will tend to be smaller than the "lower" gaps (lower meaning that i is closer to 1 than to m-1), so that e.g.,

$$(X_{m,n} - X_{m-1,n}) / (X_{2,n} - X_{1,n}) \qquad (4)$$

will tend to be smaller under the hypothesis that the sample was drawn from

an extreme-value distribution than under most appropriate alternatives (in particular, those considered in the power calculations).

We first considered using only the ratio of these extreme gaps in our test statistic, but it became apparent from power calculations that other information in the sample should be used. To include other upper and lower gaps in the numerator and denominator, respectively, of (4) we cannot simply add them because of the "telescoping" that would occur. To circumvent this, and to capitalize on the previously noted properties of the L statistic, we based our test on the ratio of sums of leaps, rather than on the ratio of sums of gaps.

In calculating the expected values appearing in the denominators of the leaps ℓ_i, we used published tables [18] of the expected values of the "reduced" extreme-value order statistics $Y_{i,n} = (X_{i,n} - u)/b$. Of course, the value of L is not affected if one uses $\ell'_j = (X_{j+1,n} - X_{j,n}) / \left[E(Y_{j+1,n}) - E(Y_{j,n})\right]$ instead of ℓ_j in equation (3). The differences $\left[E(Y_{i+1,n}) - E(Y_{i,n})\right]$ for $n = 3(1)25$, $i = 1(1)n-1$ are also given in Table A-9 herein.

Sampling studies concerning the optimum treatment of a sample censored at the <u>m</u>th of n ordered observations (optimum with respect to providing most

powerful tests) revealed that one should form L with the average of the first m/2 or (m-1)/2 (whichever is an integer) leaps in the denominator and the average of the remaining leaps in the numerator.

For convenience in calculating percentage points, we prefer to work with a statistic which takes on values only in the unit interval rather than with a statistic like L which can take on any positive value. Since L is like an F variate and since a Beta variate, Z, taking values only on the unit interval, can be obtained from an F variate with ν_1 and ν_2 degrees of freedom via the transformation

$$Z = (\nu_1/\nu_2)\, F \Big/ \left[1 + (\nu_1/\nu_2)\, F\right] ,$$

we employ a similar transformation to obtain a new statistic S. Specifically

$$S = (r/s)\, L \,/\, [1 + (r/s)\, L],$$

which yields

$$S = \sum_{j=m-r}^{m-1} \ell'_j \Big/ \sum_{j=1}^{m-1} \ell'_j .$$

As discussed above, we take $r = [m/2]$, where $[x]$ denotes the greatest integer contained in x. Thus, the statistic on which our test is based is

$$S = \frac{\sum_{i=[m/2]+1}^{m-1} (X_{i+1,n} - X_{i,n}) / \left[E\,(Y_{i+1,n}) - E\,(Y_{i,n}) \right]}{\sum_{i=1}^{m-1} (X_{i+1,n} - X_{i,n}) / \left[E\,(Y_{i+1,n}) - E\,(Y_{i,n}) \right]} \tag{5}$$

Note that S is much simpler to calculate than the statistics associated with the analogues of the classical tests given in Section 2, since the parameters u and b need not be estimated and $F\,(X_i)$ need not be calculated.

A computer program has been written to calculate and tabulate percentiles of S by means of a Monte Carlo procedure. Table A-9 contains the 100 λth percentiles of the S statistic based on samples censored at m out of n observations for λ = .75, .80, .85, .90, .95, .99; m = 3(1)n; n = 3(1)25. Differences of the expected values of the reduced order statistics from the extreme-value distribution, which are required in the calculation of S, are also given in Table A-9. Details of the computational procedure used in calculating the percentiles are given in Section 5.

4. POWER COMPARISONS

In this section, we present tables showing the power of analogues, using parameter estimates, of the (two-sided) Kolmogorov-Smirnov test, the Cramér-von Mises test, the Anderson-Darling test, the Kuiper test, and several different versions of the S test. These power calculations were obtained by Monte Carlo simulation. For the four classical tests, best linear invariant estimates of u and b were substituted for the parameters in $F_0(X_i)$. Tables for obtaining best linear invariant estimates of u and b are given in [17]; optimality properties of the estimators are derived in [21] and comparisons of expected losses of the estimators with those of other invariant estimators are given in [12] and [19].

For all the classical goodness-of-fit tests, the percentage points of the test statistics are based on Monte Carlo samples of size 15,000. For calculating percentiles of the S statistic, the Monte Carlo sample size was 20,000. For the three other statistics mentioned below (L, S', and S''), 2500 samples were generated in determining the percentiles. In each case shown below in the tabulations, 2500 samples were generated in calculating the power functions.

Our first case dealt with uncensored samples of size 5. This run was made before we had considered the S statistic and is based on an L statistic, the ratio of the last leap, ℓ_4 to the first leap, ℓ_1. Table A-1 shows the power of the test based on this statistic and of each of the above-named classical tests of the hypothesis that the sample was drawn from any two-parameter Weibull distribution against the alternative that the sample was drawn from a three-parameter Weibull distribution with $\mu/\delta = 10$, $\beta = 1$ (actually a two-parameter exponential distribution).

In Table A-1, and thereafter, n is the size of each sample, m is the censoring number, "size" heads the column giving the probability of type I error, "L" heads the column giving the power of the L test (this appears only in Tables A-1 and A-2. The symbol, S, referring to the power of the S test, appears in Tables A-3 through A-6), "D_n" heads the column giving the power of the (two-sided) Kolmogorov-Smirnov test, "ω_n^2" heads the column giving the power of the Cramér-von Mises test, "W_n" heads the column giving the power of the Anderson-Darling test, and "V_n" heads the column giving the power of the Kuiper test. Note that, except for one entry, the L test gives better power than the other tests in Table A-1.

TABLE A-1

Power of Several Tests of Two-Parameter Weibull vs Three-Parameter Weibull (with μ/δ = 10, β = 1) n = m = 5

size	L	D_n	ω_n^2	W_n	V_n
.01	.07	.09	.10	.06	.09
.05	.27	.24	.27	.21	.23
.10	.44	37	.40	.33	.35
.15	.54	46	.49	.43	.45
.20	.63	.54	.56	.50	.50
.25	.70	.60	.61	.56	.56

Table A-2 shows the power of the various tests of the hypothesis that the data were drawn from an extreme-value distribution against the alternative that they were drawn from a normal distribution for n = m = 5. This can also be viewed as a test of the hypothesis that the data are (two-parameter) Weibull vs the alternative that they are from a log-normal distribution, one sometimes used in survival analysis. Again, while the L test is not uniformly best, it does show up well. It appears from Tables A-1 and A-2 that the relative merit of the L test increases with the size of the test.

TABLE A-2

Power of Several Tests of Extreme-Value vs Normal
(or Two-Parameter Weibull vs Log-Normal). $n = m = 5$

size	L	D_n	ω_n^2	W_n	V_n
0.01	0.01	0.02	0.01	0.01	0.02
0.05	0.07	0.07	0.07	0.06	0.07
0.10	0.15	0.15	0.15	0.12	0.13
0.15	0.22	0.21	0.20	0.16	0.19
0.20	0.29	0.27	0.26	0.22	0.23
0.25	0.36	0.33	0.31	0.27	0.27

At this point of the investigation, it was determined that a better test could be obtained by considering more than merely the extreme leaps. Thus, the next case we calculated involved $m = n = 5$ and $S = (\ell'_3 + \ell'_4) / \sum_1^4 \ell'_j$.

We see from Tables A-3 and A-4 that the S test is more powerful than the L test, used in Tables 1 and 2, for small probabilities of type I error (size) and thus compares more favorably in these cases with the other tests. Tables A-3 and A-4 give the powers of the same hypothesis-alternative pairs as Tables A-1 and A-2, respectively. Note that there is some discrepancy in power of the analogues of the classical tests in Tables A-2 and A-4 because only 2500 samples were used in the power calculations.

TABLE A-3

Power of Several Tests of Two-Parameter Weibull vs Three-Parameter Weibull (with $\mu/\delta=10$, $\beta=1$) n=m=5

size	S	D_n	ω_n^2	W_n	V_n
0.01	0.10	0.09	0.10	0.06	0.09
0.05	0.30	0.24	0.27	0.21	0.23
0.10	0.44	0.37	0.40	0.33	0.35
0.15	0.54	0.46	0.49	0.43	0.45
0.20	0.62	0.54	0.56	0.50	0.50
0.25	0.68	0.60	0.61	0.56	0.56

TABLE A-4

Power of Several Tests of Extreme-Value vs Normal (or Two-Parameter Weibull vs Log-Normal). n=m=5

size	S	D_n	ω_n^2	W_n	V_n
0.01	0.02	0.02	0.02	0.01	0.01
0.05	0.09	0.08	0.08	0.05	0.06
0.10	0.17	0.16	0.16	0.12	0.14
0.15	0.25	0.23	0.22	0.18	0.20
0.20	0.31	0.29	0.28	0.24	0.25
0.25	0.38	0.35	0.33	0.29	0.31

In order to assure ourselves that the observed results extend to more general three-parameter Weibull models, we calculated power values for the analogues of the classical tests, the L test and the S test for $n=m=5$, $\mu/\delta = 10$, $\beta = .5$. For each test for a given probability of type I error, the power value is considerably larger than the value shown in Table A-1 and/or Table A-3. In general, however, the power relationships between tests for a fixed probability of type I error did not change appreciably from those exhibited in the tables corresponding to $n=m=5$, $\mu/\delta = 10$, $\beta = 1$.

Next we increased the sample size from 5 to 10, still with no censoring. Note in Tables A-5 and A-6 that the S test is at least as powerful as the others and, in some cases, considerably more powerful. In these tables $S = \sum_{j=6}^{9} \ell'_j \Big/ \sum_{j=1}^{9} \ell'_j$.

TABLE A-5

Power of Several Tests of Two-Parameter Weibull vs Three-Parameter Weibull (with $\mu/\delta=10$, $\beta=1$) $n=m=10$

size	S	D_n	ω_n^2	W_n	V_n
0.01	0.37	0.21	0.34	0.31	0.30
0.05	0.64	0.45	0.59	0.56	0.56
0.10	0.75	0.61	0.71	0.69	0.67
0.15	0.82	0.69	0.79	0.77	0.74
0.20	0.85	0.76	0.83	0.82	0.79
0.25	0.88	0.80	0.86	0.85	0.83

TABLE A-6

Power of Several Tests of Extreme-Value vs Normal (or Two-Parameter Weibull vs Log-Normal). n=m=10

size	S	D_n	ω_n^2	W_n	V_n
0.01	0.04	0.03	0.04	0.03	0.02
0.05	0.17	0.12	0.12	0.10	0.10
0.10	0.29	0.20	0.21	0.19	0.17
0.15	0.38	0.27	0.30	0.25	0.25
0.20	0.47	0.35	0.36	0.32	0.31
0.25	0.54	0.40	0.43	0.37	0.37

The numerical investigations summarized in Tables A-3 through A-6 show the desirable power properties of the S test. Its only serious competitor is the Cramer-von Mises test. Because it is considerably more difficult to compute the test statistic, ω_n^2, than the S statistic, one would probably prefer the S test to the Cramer-vcn Mises even if they had identical power.

The remaining two tables in this section verify our conjecture that the S test is at least as powerful if more upper leaps are included in its numerator. For the specified three-parameter Weibull alternative, including more upper leaps increases the power considerably. Censored samples are now considered.

In Tables A-7 and A-8, we summarize power calculations for samples of size 10 censored at the seventh smallest observation. Three S statistics are considered $S = \sum_{4}^{6} \ell'_j / \sum_{1}^{6} \ell_j$, $S' = \ell'_6 / \sum_{1}^{6} \ell'_j$, and $S'' = (\ell'_5 + \ell'_6)/ \sum_{1}^{6} \ell'_j$. In the test of Two-Parameter Weibull vs Three-Parameter Weibull with $\mu/\delta = 10$, $\beta = 1$ (Table A-7) the S statistic is shown always to give a more powerful test. In the case of a Gaussian alternative (Table A-8), S and S" seem to be comparable.

TABLE A-7

Power of Three Tests Based on S Statistics of Two-Parameter Weibull vs Three-Parameter Weibull (with $\mu/\delta=10$, $\beta=1$) n=10, m=7

size	S"	S'	S
0.01	0.14	0.09	0.17
0.05	0.32	0.23	0.38
0.10	0.45	0.34	0.50
0.15	0.54	0.41	0.61
0.20	0.59	0.48	0.68
0.25	0.65	0.53	0.73

TABLE A-8

Power of Three Tests Based on S Statistics of Extreme-Value vs Normal (or Two-Parameter Weibull vs Log-Normal) n=10, m=7

size	S"	S'	S
0.01	0.03	0.02	0.03
0.05	0.10	0.10	0.10
0.10	0.19	0.14	0.17
0.15	0.26	0.26	0.26
0.20	0.32	0.31	0.32
0.25	0.40	0.37	0.40

After the investigations described in this paper had been completed, a test described by Gnedenko, et al [10] for constant hazard rate versus increasing or decreasing hazard rate came to the attention of the authors. The statistic associated with this test is (for T_0 defined to be zero and T_i, i=1, ..., m, the ith smallest observed failure time)

$$(m-r_1) \sum_{i=1}^{r_1} (n-i+1)\,(T_i-T_{i-1}) \Big/ \left[r_1 \sum_{i=r_1+1}^{m} (n-i+1)\,(T_i-T_{i-1}) \right]$$

which, under the hypothesis of constant hazard rate, is distributed as F with 2r and $2(m-r_1)$ degrees of freedom. Monte Carlo investigations of

Fercho and Ringer [8] show Gnedenko's test with $r_1 = m/2$ or $(m-1)/2$ to be most powerful, in general, among four tests for constant hazard rate which they studied for testing exponentiality versus various two-parameter Weibull alternatives.

5. COMPUTATIONAL DETAILS

The <u>100 λ</u> th percentile y_λ of a random variable Y with cumulative distribution function F and density function f is defined by

$$F\,(y_\lambda) = \int_{-\infty}^{y_\lambda} f\,(y)\,dy = \lambda$$

If $Y_1 \leq \cdot \cdot \cdot \leq Y_N$ is a set of ordered observations of this random variable, then an appropriate estimator for y_λ is $Y_{\lambda N}$ if λN is an integer or $Y_{[\lambda N]+1}$ otherwise. It is known [23] that if $f(y_\lambda) \neq 0$ and if f is differentiable in the neighborhood of y_λ, then $Y_{\lambda N}$ is asymptotically normally distributed with mean y_λ and variance $\lambda\,(1-\lambda)\,/\,\left\{N\left[f(y_\lambda)\right]^2\right\}$.

To calculate percentiles of S (Eq. (5)), we generated N = 20,000 samples of each sample size n = 3(1)25. (The number 20,000 was chosen on the following basis. If the distribution of S were actually a Beta distribution, then using the asymptotic variance of $Y_{\lambda N}$ for the "worst" case, λ = .99, an N of approximately 20,000 would be needed to give at least 2-significant-digit accuracy in the percentile.) Because of storage limitations, we did not rank and store all N values of S, but rather we divided the unit interval into 300 subintervals of equal length and counted the

number of S's that fell into each subinterval. If T_k is the number of S's that fall into the kth subinterval and if J is such that

$$\sum_{k=1}^{J-1} T_k < \lambda N \leq \sum_{k=1}^{J} T_k$$

then we estimate $S_{\lambda N}$, and thereby the 100 λth percentile of the distribution of S, by

$$(J-1)/300 + (\lambda N - \sum_{k=1}^{J-1} T_k) / (300\ T_J) .$$

This will always be within 1/300 of $S_{\lambda N}$ and often closer because of the second-order correcting term. (In fact, it follows from the standard formula for error of interpolation (see, e.g., Ralston [24 , p. 60]) that the absolute error is bounded by $(1/600)^2\ f'(\xi) / f(\xi)$ where ξ lies in the same subinterval as y_λ.)

For a fixed sample size n, we used the same N values of S to obtain percentiles for each censoring number m = 3(1)n. We generated a new batch of N S's for each different value of n, n = 3(1)25.

The reduced extreme-value variates $Y_{i,n}$ which are needed to calculate S were obtained via $Y = \ln[\ln (1/V)]$, where V is a uniform (0,1) variate; the variates $R_{i,n}$ used in power calculations against the alternative $\mu/\delta = 10$, $\beta = 1, .5$ were obtained via $R = \left[\mu/\delta + \ln (1/V)^{1/\beta}\right]^{\beta}$ with $\mu/\delta = 10$, $\beta = 1, .5$. We use R in power calculations when we consider the case where, in obtaining an extreme-value variate by taking the natural logarithm of a Weibull variate T, one incorrectly assumes μ to be zero. More values of $Y_{i,n}$ than of $\ell n\, R_{i,n}$ lie to the left of the mean of the distribution of $\ell n\, R_{i,n}$ so that the tail properties of the test based on S contribute to its power. Normal variates used in power calculations were generated by the method of Marsaglia and Bray [22]. Independent samples of the various random variables were obtained in each repetition.

Finally, we mention that while we were generating samples of reduced extreme-value variates to obtain the percentiles of S, we also took the opportunity to obtain percentiles of the distributions of best linear invariant estimates of the location and scale parameters and of certain percentage points of the extreme-value distribution. Tabulations of these percentiles appear in [21].

6. TABLES AND EXAMPLE

Below are the tables to be used in connection with the test based on the statistic S. An example illustrating their use follows the tables.

TABLE A-9

Percentiles of the Distribution of S and Differences of Expected Values of Reduced Extreme-Value Order Statistics

n	m	$EY_{m+1}-EY_m$	0.75	0.80	0.85	0.90	0.95	0.99
3	1	1.216395						
	2	0.863046						
	3		0.75	0.79	0.84	0.90	0.95	0.99
4	1	1.150727						
	2	0.706698						
	3	0.679596	0.74	0.79	0.85	0.90	0.95	0.99
	4		0.50	0.55	0.60	0.67	0.76	0.89
5	1	1.115718						
	2	0.645384						
	3	0.532445	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.583273	0.50	0.56	0.61	0.68	0.77	0.89
	5		0.67	0.71	0.75	0.79	0.86	0.94
6	1	1.093929						
	2	0.612330						
	3	0.474330	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.442920	0.50	0.55	0.61	0.68	0.76	0.89
	5	0.522759	0.67	0.71	0.75	0.80	0.86	0.93
	6		0.54	0.57	0.61	0.66	0.73	0.84
7	1	1.079055						
	2	0.591587						
	3	0.442789	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.387289	0.50	0.55	0.61	0.68	0.77	0.89
	5	0.387714	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.480648	0.54	0.58	0.62	0.67	0.74	0.85
	7		0.64	0.67	0.70	0.74	0.80	0.88
8	1	1.068252						
	2	0.577339						
	3	0.422889	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.356967	0.50	0.55	0.61	0.68	0.77	0.90
	5	0.334089	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.349907	0.54	0.58	0.62	0.67	0.74	0.85
	7	0.449338	0.64	0.67	0.70	0.74	0.80	0.89
	8		0.55	0.58	0.61	0.65	0.71	0.81

TABLE A-9 - (Continued)

Percentiles of the Distribution of S and Differences of Expected Values of Reduced Extreme-Value Order Statistics

n	m	$EY_{m+1}-EY_m$	0.75	0.80	0.85	0.90	0.95	0.99
9	1	1.060046						
	2	0.566942						
	3	0.4091 7	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.337763	0.50	0.55	0.61	0.68	0.77	0.89
	5	0.304777	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.297949	0.54	0.58	0.62	0.67	0.75	0.86
	7	0.322189	0.63	0.67	0.70	0.74	0.80	0.89
	8	0.424958	0.55	0.58	0.61	0.66	0.72	0.82
	9		0.62	0.64	0.67	0.71	0.76	0.85
10	1	1.053606						
	2	0.559013						
	3	0.399100	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.324470	0.50	0.55	0.61	0.68	0.77	0.90
	5	0.286163	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.269493	0.54	0.58	0.62	0.68	0.75	0.85
	7	0.271645	0.63	0.67	0.71	0.75	0.81	0.89
	8	0.300869	0.55	0.58	0.62	0.66	0.72	0.81
	9	0.405316	0.62	0.65	0.68	0.71	0.76	0.85
	10		0.55	0.58	0.61	0.64	0.69	0.79
11	1	1.048411						
	2	0.552769						
	3	0.391410	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.314705	0.49	0.55	0.61	0.68	0.77	0.90
	5	0.273745	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.251386	0.54	0.58	0.63	0.68	0.75	0.86
	7	0.243928	0.64	0.67	0.71	0.75	0.81	0.89
	8	0.251548	0.55	0.58	0.62	0.66	0.72	0.82
	9	0.283879	0.62	0.64	0.68	0.71	0.77	0.85
	10	0.389071	0.55	0.58	0.61	0.64	0.70	0.79
	11		0.60	0.63	0.65	0.69	0.74	0.82
12	1	1.044137						
	2	0.547721						
	3	0.385338	0.75	0.79	0.84	0.90	0.95	0.99
	4	0.307221	0.50	0.55	0.61	0.68	0.78	0.89
	5	0.263737	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.238797	0.54	0.58	0.62	0.67	0.74	0.85
	7	0.226264	0.64	0.67	0.70	0.75	0.81	0.89
	8	0.224477	0.55	0.58	0.62	0.66	0.72	0.82
	9	0.235630	0.62	0.64	0.68	0.71	0.77	0.85
	10	0.269966	0.55	0.58	0.61	0.65	0.70	0.79
	11	0.375356	0.60	0.63	0.66	0.69	0.74	0.82
	12		0.55	0.57	0.60	0.63	0.68	0.76
13	1	1.040555						
	2	0.543556						
	3	0.380417	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.301300	0.50	0.55	0.61	0.68	0.77	0.89
	5	0.256437	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.229515	0.54	0.58	0.63	0.68	0.75	0.86
	7	0.213966	0.64	0.67	0.71	0.75	0.81	0.90
	8	0.207205	0.55	0.58	0.62	0.66	0.72	0.82
	9	0.209131	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.222667	0.55	0.58	0.61	0.65	0.70	0.79
	11	0.258323	0.60	0.63	0.66	0.69	0.74	0.82
	12	0.363582	0.55	0.57	0.60	0.64	0.68	0.76
	13		0.59	0.61	0.64	0.67	0.72	0.79

TABLE A-9 - (Continued)

Percentiles of the Distribution of S and Differences of Expected Values of Reduced Extreme-Value Order Statistics

n	m	$EY_{m+1}-EY_m$	0.75	0.80	0.85	0.90	0.95	0.99
14	1	1.037513						
	2	0.540059						
	3	0.376352	0.75	0.79	0.85	0.90	0.95	0.99
	4	0.296496	0.49	0.54	0.61	0.68	0.77	0.90
	5	0.250650	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.222377	0.54	0.58	0.62	0.68	0.74	0.86
	7	0.204885	0.64	0.67	0.71	0.75	0.81	0.89
	8	0.195165	0.55	0.58	0.62	0.66	0.73	0.82
	9	0.192209	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.196679	0.55	0.58	0.61	0.65	0.70	0.79
	11	0.211875	0.60	0.63	0.66	0.69	0.74	0.82
	12	0.248409	0.55	0.57	0.60	0.64	0.68	0.77
	13	0.353334	0.59	0.61	0.64	0.67	0.72	0.79
	14		0.55	0.57	0.59	0.62	0.67	0.75
15	1	1.034894						
	2	0.537085						
	3	0.372934	0.75	0.80	0.84	0.90	0.95	0.99
	4	0.292518	0.51	0.56	0.62	0.69	0.78	0.90
	5	0.245947	0.68	0.71	0.76	0.80	0.86	0.94
	6	0.216712	0.54	0.58	0.62	0.67	0.75	0.86
	7	0.197893	0.64	0.67	0.71	0.75	0.81	0.89
	8	0.186266	0.55	0.58	0.62	0.66	0.72	0.82
	9	0.180402	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.180072	0.55	0.58	0.61	0.65	0.70	0.79
	11	0.186347	0.61	0.63	0.66	0.69	0.74	0.82
	12	0.202727	0.55	0.57	0.60	0.64	0.68	0.77
	13	0.239842	0.59	0.62	0.64	0.67	0.72	0.79
	14	0.344309	0.55	0.57	0.60	0.63	0.67	0.75
	15		0.59	0.61	0.63	0.66	0.70	0.77
16	1	1.032617						
	2	0.534521						
	3	0.370021	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.289169	0.51	0.56	0.62	0.69	0.78	0.89
	5	0.242049	0.68	0.72	0.76	0.80	0.86	0.94
	6	0.212103	0.54	0.58	0.63	0.68	0.75	0.86
	7	0.192338	0.64	0.67	0.71	0.75	0.81	0.89
	8	0.179407	0.55	0.58	0.62	0.66	0.72	0.82
	9	0.171667	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.168476	0.55	0.58	0.61	0.65	0.71	0.79
	11	0.170026	0.60	0.63	0.66	0.69	0.74	0.82
	12	0.177619	0.55	0.58	0.60	0.64	0.69	0.77
	13	0.194859	0.60	0.62	0.64	0.68	0.72	0.80
	14	0.232350	0.55	0.57	0.60	0.63	0.67	0.75
	15	0.336283	0.59	0.61	0.63	0.66	0.70	0.77
	16		0.55	0.57	0.59	0.62	0.66	0.73

TABLE A-9 - (Continued)

Percentiles of the Distribution of S and Differences of Expected Values of Reduced Extreme-Value Order Statistics

n	m	$EY_{m+1}-EY_m$	0.75	0.80	0.85	0.90	0.95	0.99
17	1	1.030618						
	2	0.532290						
	3	0.367507	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.286312	0.50	0.55	0.61	0.69	0.78	0.90
	5	0.238765	0.67	0.71	0.75	0.80	0.87	0.94
	6	0.208278	0.54	0.58	0.62	0.68	0.74	0.85
	7	0.187813	0.64	0.67	0.71	0.75	0.80	0.89
	8	0.173951	0.55	0.58	0.62	0.66	0.72	0.81
	9	0.164928	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.159891	0.55	0.58	0.61	0.65	0.70	0.79
	11	0.158624	0.61	0.63	0.66	0.69	0.74	0.82
	12	0.161559	0.55	0.58	0.61	0.64	0.69	0.77
	13	0.170132	0.59	0.62	0.64	0.67	0.72	0.80
	14	0.188005	0.55	0.57	0.60	0.63	0.68	0.75
	15	0.225729	0.59	0.61	0.63	0.66	0.70	0.77
	16	0.329085	0.55	0.57	0.59	0.62	0.66	0.74
	17		0.58	0.60	0.62	0.65	0.69	0.75
18	1	1.028850						
	2	0.530332						
	3	0.365314	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.283846	0.49	0.55	0.61	0.68	0.77	0.90
	5	0.235958	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.205051	0.54	0.58	0.62	0.67	0.75	0.86
	7	0.184055	0.64	0.67	0.71	0.75	0.81	0.89
	8	0.169504	0.55	0.58	0.62	0.66	0.73	0.82
	9	0.159564	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.153763	0.55	0.58	0.61	0.65	0.71	0.80
	11	0.150176	0.61	0.63	0.66	0.69	0.74	0.82
	12	0.150333	0.55	0.58	0.61	0.64	0.69	0.77
	13	0.154313	0.60	0.62	0.64	0.68	0.72	0.80
	14	0.163630	0.55	0.57	0.60	0.63	0.67	0.76
	15	0.181971	0.59	0.61	0.63	0.66	0.70	0.78
	16	0.219825	0.55	0.57	0.59	0.62	0.66	0.74
	17	0.322580	0.58	0.60	0.62	0.65	0.69	0.76
	18		0.55	0.57	0.59	0.61	0.65	0.72
19	1	1.027277						
	2	0.528594						
	3	0.363389	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.281692	0.50	0.55	0.61	0.69	0.78	0.90
	5	0.233535	0.67	0.71	0.76	0.81	0.86	0.94
	6	0.202291	0.54	0.58	0.62	0.68	0.75	0.86
	7	0.180882	0.64	0.67	0.71	0.75	0.81	0.89
	8	0.165807	0.55	0.58	0.62	0.67	0.72	0.82
	9	0.155189	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.147984	0.55	0.58	0.61	0.65	0.71	0.80
	11	0.143650	0.61	0.63	0.66	0.69	0.74	0.82
	12	0.142012	0.55	0.58	0.60	0.64	0.69	0.77
	13	0.143250	0.60	0.62	0.64	0.68	0.72	0.80
	14	0.148031	0.55	0.58	0.60	0.63	0.68	0.76
	15	0.157921	0.59	0.61	0.63	0.66	0.70	0.78
	16	0.176611	0.55	0.57	0.59	0.62	0.66	0.74
	17	0.214520	0.58	0.60	0.62	0.65	0.69	0.76
	18	0.316666	0.55	0.57	0.59	0.61	0.65	0.72
	19		0.57	0.59	0.61	0.64	0.67	0.74

TABLE A-9 - (Continued)

Percentiles of the Distribution of S and Differences of Expected Values of Reduced Extreme-Value Order Statistics

n	m	$EY_{m+1}-EY_m$	0.75	0.80	0.85	0.90	0.95	0.99
20	1	1.025866						
	2	0.527046						
	3	0.361682	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.279798	0.50	0.55	0.61	0.68	0.78	0.90
	5	0.231417	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.199905	0.54	0.58	0.62	0.67	0.75	0.86
	7	0.178167	0.64	0.67	0.71	0.75	0.81	0.89
	8	0.162684	0.55	0.58	0.62	0.66	0.73	0.82
	9	0.151549	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.143674	0.55	0.58	0.61	0.65	0.71	0.80
	11	0.138448	0.61	0.63	0.66	0.69	0.74	0.83
	12	0.135580	0.55	0.58	0.61	0.64	0.69	0.77
	13	0.135046	0.60	0.62	0.65	0.68	0.72	0.80
	14	0.137120	0.55	0.57	0.60	0.63	0.68	0.76
	15	0.142527	0.59	0.61	0.63	0.66	0.71	0.78
	16	0.152861	0.55	0.57	0.59	0.62	0.67	0.74
	17	0.171810	0.58	0.60	0.62	0.65	0.69	0.76
	18	0.209721	0.55	0.57	0.59	0.62	0.66	0.72
	19	0.311257	0.58	0.59	0.61	0.64	0.68	0.74
	20		0.55	0.56	0.58	0.61	0.65	0.71
21	1	1.024594						
	2	0.525657						
	3	0.360159	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.278117	0.50	0.56	0.62	0.69	0.78	0.90
	5	0.229551	0.68	0.71	0.76	0.80	0.86	0.94
	6	0.197821	0.54	0.58	0.67	0.67	0.74	0.85
	7	0.175815	0.64	0.67	0.71	0.75	0.80	0.89
	8	0.160009	0.55	0.58	0.62	0.66	0.73	0.82
	9	0.148471	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.140087	0.55	0.58	0.61	0.65	0.70	0.80
	11	0.134200	0.60	0.63	0.66	0.69	0.74	0.82
	12	0.130451	0.55	0.58	0.60	0.64	0.69	0.77
	13	0.128702	0.59	0.62	0.64	0.68	0.72	0.79
	14	0.129025	0.55	0.57	0.60	0.63	0.67	0.75
	15	0.131756	0.59	0.61	0.63	0.66	0.70	0.78
	16	0.137659	0.55	0.57	0.60	0.63	0.67	0.74
	17	0.148341	0.58	0.60	0.62	0.65	0.69	0.76
	18	0.167481	0.55	0.57	0.59	0.62	0.66	0.73
	19	0.205352	0.58	0.60	0.62	0.64	0.68	0.75
	20	0.306285	0.55	0.56	0.58	0.61	0.65	0.72
	21		0.57	0.59	0.61	0.63	0.67	0.73

TABLE A-9 - (Continued)

Percentiles of the Distribution of S and Differences of Expected Values of Reduced Extreme-Value Order Statistics

n	m	$EY_{m+1}-EY_m$	0.75	0.80	0.85	0.90	0.95	0.99
22	1	1.023439						
	2	0.524405						
	3	0.358790	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.276618	0.50	0.55	0.61	0.68	0.77	0.90
	5	0.227895	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.195983	0.54	0.58	0.63	0.68	0.75	0.85
	7	0.173760	0.64	0.67	0.71	0.75	0.81	0.89
	8	0.157692	0.55	0.58	0.62	0.66	0.72	0.82
	9	0.145834	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.137052	0.55	0.58	0.61	0.65	0.70	0.80
	11	0.130662	0.61	0.63	0.66	0.69	0.74	0.82
	12	0.126260	0.55	0.58	0.61	0.64	0.69	0.78
	13	0.123640	0.60	0.62	0.65	0.68	0.72	0.80
	14	0.122763	0.55	0.58	0.60	0.63	0.68	0.75
	15	0.123763	0.59	0.61	0.63	0.67	0.71	0.78
	16	0.127019	0.55	0.57	0.60	0.62	0.67	0.74
	17	0.133316	0.58	0.60	0.62	0.65	0.69	0.76
	18	0.144273	0.55	0.57	0.59	0.62	0.66	0.73
	19	0.163552	0.58	0.60	0.62	0.64	0.68	0.75
	20	0.201355	0.55	0.57	0.59	0.61	0.65	0.72
	21	0.301693	0.57	0.59	0.61	0.64	0.67	0.73
	22		0.54	0.56	0.58	0.61	0.64	0.70
23	1	1.022389						
	2	0.523269						
	3	0.357557	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.275268	0.50	0.55	0.61	0.68	0.77	0.89
	5	0.226417	0.67	0.71	0.75	0.80	0.86	0.94
	6	0.194351	0.55	0.59	0.63	0.68	0.76	0.86
	7	0.171948	0.64	0.68	0.71	0.76	0.82	0.89
	8	0.155666	0.56	0.59	0.63	0.67	0.73	0.83
	9	0.143549	0.62	0.65	0.68	0.72	0.78	0.86
	10	0.134451	0.56	0.59	0.62	0.66	0.71	0.80
	11	0.127667	0.61	0.63	0.66	0.70	0.75	0.82
	12	0.122768	0.55	0.58	0.61	0.64	0.69	0.78
	13	0.119503	0.60	0.62	0.65	0.68	0.73	0.80
	14	0.117764	0.55	0.57	0.60	0.63	0.68	0.76
	15	0.117577	0.59	0.61	0.63	0.67	0.71	0.78
	16	0.119120	0.55	0.57	0.60	0.63	0.67	0.75
	17	0.122799	0.58	0.60	0.63	0.65	0.69	0.77
	18	0.129416	0.55	0.57	0.59	0.62	0.66	0.73
	19	0.140590	0.58	0.60	0.62	0.64	0.68	0.75
	20	0.159966	0.55	0.57	0.59	0.61	0.65	0.72
	21	0.197679	0.57	0.59	0.61	0.63	0.67	0.73
	22	0.297435	0.55	0.56	0.58	0.60	0.64	0.70
	23		0.57	0.58	0.60	0.63	0.66	0.72

TABLE A-9 - (Concluded)

Percentiles of the Distribution of S and Differences of Expected Values of Reduced Extreme-Value Order Statistics

n	m	$EY_{m+1}-EY_m$	0.75	0.80	0.85	0.90	0.95	0.99
24	1	1.021431						
	2	0.522233						
	3	0.356436	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.274051	0.50	0.56	0.62	0.69	0.78	0.90
	5	0.225086	0.67	0.71	0.76	0.81	0.86	0.94
	6	0.192892	0.54	0.58	0.62	0.68	0.75	0.85
	7	0.170338	0.64	0.67	0.71	0.75	0.81	0.89
	8	0.153877	0.55	0.58	0.62	0.67	0.73	0.83
	9	0.141549	0.62	0.65	0.68	0.72	0.77	0.86
	10	0.132195	0.56	0.58	0.61	0.66	0.71	0.80
	11	0.125099	0.61	0.63	0.66	0.70	0.75	0.83
	12	0.119811	0.55	0.58	0.61	0.64	0.70	0.78
	13	0.116054	0.60	0.62	0.65	0.68	0.73	0.80
	14	0.113677	0.55	0.58	0.60	0.64	0.68	0.76
	15	0.112638	0.59	0.61	0.64	0.67	0.71	0.78
	16	0.113007	0.55	0.57	0.60	0.63	0.67	0.74
	17	0.114990	0.58	0.60	0.62	0.65	0.69	0.76
	18	0.119014	0.55	0.57	0.59	0.62	0.66	0.73
	19	0.125889	0.58	0.60	0.62	0.64	0.68	0.75
	20	0.137235	0.55	0.57	0.59	0.61	0.65	0.72
	21	0.156679	0.57	0.59	0.61	0.64	0.67	0.73
	22	0.194285	0.55	0.56	0.58	0.61	0.64	0.71
	23	0.293473	0.57	0.59	0.60	0.63	0.66	0.72
	24		0.54	0.56	0.58	0.60	0.64	0.69
25	1	1.020551						
	2	0.521285						
	3	0.355415	0.75	0.80	0.85	0.90	0.95	0.99
	4	0.272945	0.50	0.56	0.61	0.69	0.78	0.91
	5	0.223885	0.67	0.71	0.76	0.81	0.87	0.94
	6	0.191578	0.54	0.58	0.62	0.68	0.75	0.86
	7	0.168899	0.64	0.67	0.71	0.75	0.81	0.89
	8	0.152286	0.55	0.58	0.62	0.66	0.72	0.82
	9	0.139783	0.62	0.65	0.68	0.72	0.77	0.85
	10	0.130219	0.56	0.58	0.61	0.65	0.71	0.80
	11	0.122871	0.61	0.63	0.66	0.70	0.75	0.82
	12	0.117274	0.55	0.58	0.61	0.64	0.69	0.78
	13	0.113132	0.60	0.62	0.65	0.68	0.73	0.81
	14	0.110268	0.55	0.58	0.60	0.63	0.68	0.76
	15	0.108598	0.59	0.61	0.64	0.66	0.71	0.78
	16	0.108124	0.55	0.57	0.60	0.63	0.67	0.74
	17	0.108944	0.58	0.60	0.62	0.65	0.69	0.76
	18	0.111289	0.55	0.57	0.59	0.62	0.66	0.73
	19	0.115596	0.58	0.60	0.62	0.64	0.68	0.75
	20	0.122683	0.55	0.57	0.59	0.61	0.65	0.72
	21	0.134165	0.57	0.59	0.61	0.63	0.67	0.74
	22	0.153650	0.55	0.56	0.58	0.61	0.64	0.71
	23	0.191137	0.57	0.58	0.60	0.63	0.66	0.72
	24	0.289773	0.54	0.56	0.58	0.60	0.63	0.70
	25		0.56	0.58	0.60	0.62	0.65	0.71

Example: Listed below are ordered observations $t_{i,22}$ from a sample (ignition times) of size 22, censored at the 15th smallest observation, i.e., n = 22, m = 15. We wish to test the hypothesis that this sample was drawn from a two-parameter Weibull distribution, with a three-parameter Weibull an appropriate alternative. Accordingly, we first form $\ell_i = (x_{i+1,22} - x_{i,22}) / \left[E(Y_{i+1,22}) - E(Y_{i,22})\right]$ for i = 1, ... , 14, where $x_{i,22} = \ell n\,(t_{i,22})$, and the $E(Y_{i+1,n}) - E(Y_{i,n})$ are read from Table A-9. We then compute $S = \sum_{8}^{14} \ell_i' \Big/ \sum_{1}^{14} \ell_i'$.

For the data:

$t_{1,22}$ =	15.5	$t_{6,22}$ =	20.6	$t_{11,22}$ =	26.5
$t_{2,22}$ =	15.6	$t_{7,22}$ =	22.8	$t_{12,22}$ =	26.5
$t_{3,22}$ =	16.5	$t_{8,22}$ =	23.1	$t_{13,22}$ =	32.7
$t_{4,22}$ =	17.5	$t_{9,22}$ =	23.5	$t_{14,22}$ =	33.8
$t_{5,22}$ =	19.5	$t_{10,22}$ =	24.5	$t_{15,22}$ =	33.9

we calculate S = 0.660 and therefore the hypothesis that these data were drawn from a two-parameter Weibull distribution is rejected at the 15 percent significance level. (One rejects the hypothesis if the calculated value of S <u>exceeds</u> its tabulated percentile at 1 minus the appropriate level of significance).

If one is testing the hypothesis that a sample was drawn from an extreme-value distribution, he calculates the ℓ_i''s directly <u>without</u> first taking the natural logarithms of the ordered observations.

We note that whenever m is equal to 3, the numerator of the S statistic consists of the first leap and the denominator of the first two leaps. From tabulated values, it can be observed that for m equal to 3, the distribution of S is essentially uniform, i.e., Beta with parameters 1 and 1. In other words, the ratio of the first and second leaps has an F distribution with 2 and 2 degrees of freedom. This was also observed to be true for the ratio of the first and fourth leaps for n = m = 5.

These facts suggest that the ratio of the <u>m-1</u>st leap and the sum of the <u>m-1</u>st and the first leaps apparently has very nearly a uniform distribution. The test based on this ratio appears to be fairly powerful if its size is not too small. One can, therefore, very easily test the fit of any set of observations with $n \leq 100$ to the extreme value distribution by using tables of White [27] which give the expected values of reduced extreme-value order statistics for n = 1(1)100.

If, for fixed m and λ, the tabulated percentile values are compared with percentile values of appropriate Beta distributions [with parameters

$(m-1)/2$ and $(m-1)/2$ or $(m-2)/2$ and $m/2]$, it can be seen that as sample size n increases the two tabulated values tend to agree. In fact, there is little discrepancy, even for small n. This suggests that the test for fit based on the S statistic can be used for samples of size as large as one hundred by making use of the tables of expected values of the reduced extreme-value order statistics (see White [27]) and tables of percentiles of the Beta distribution (see, for example, Harter [11]). One can also use tables of the F distribution (with m-1 and m-1 or m-2 and m degrees of freedom) with the L statistic described in Section 3.

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B. TABLES FOR OBTAINING CONFIDENCE BOUNDS AND TOLERANCE BOUNDS BASED ON BEST LINEAR INVARIANT ESTIMATES OF PARAMETERS OF THE EXTREME-VALUE DISTRIBUTION

SUMMARY

Tables are given for obtaining confidence bounds for the two parameters and the 90th, 95th, and 99th percentiles of the two-parameter Weibull or extreme-value distributions. The tables are based on best linear invariant estimators of extreme-value location and scale parameters and apply to samples of size n, n = 3(1)25, which may be censored at the mth smallest sample observation, m = 3(1)n. Discussion is given concerning other methods of obtaining confidence and tolerance bounds for these distributions, properties of the estimators on which the bounds are based and computational procedures used.

1. INTRODUCTION

In the following, we consider the problem of obtaining small-sample confidence and tolerance bounds based on censored samples from the two-parameter Weibull or, equivalently, the extreme-value distribution. A random variate T is said to have a two-parameter Weibull distribution if

$$P\left[T \leq t\right] = \begin{cases} 1 - \exp\left[-(t/\delta)^{\beta}\right], & t \geq 0 \\ 0 \quad , & t < 0 \; . \end{cases} \tag{1}$$

Both the scale parameter δ and the shape parameter β of the distribution are positive. If one makes the transformation $X = \ell n\ T$, then

$$P\left[X \leq x\right] = 1-\exp\left\{-\exp\left[(x-u)/b\right]\right\} \quad , \tag{2}$$

with $u = \ell n\ \delta$ and $b = (1/\beta) > 0$.

The two-parameter Weibull distribution has many applications (see [8] and [16]), but it is most often applied to problems of reliability or survival analysis. The reliability function $R\ (t_m)$, which gives the probability of survival at least until a "mission" time t_m, is equal to $P\left[T > t_m\right]$. The function $R\ (t_m) = \exp\left[-(t_m/\delta)^{\beta}\right]$ also can be thought of as the proportion

of the population (from which a sample is selected) surviving at time t_m. Thus, if a survival proportion γ is specified, then the time t_γ associated with this survival proportion is equal to $\delta\left[\ell n(1/\gamma)\right]^b$ and $x_\gamma = \ell n\ t_\gamma$ is equal to $u + b\ \ell n\left[\ell n\ (1/\gamma)\right]$. The time t_γ is sometimes referred to as the reliable life associated with the survival proportion γ.

The quantities t_γ and x_γ are actually percentiles of the distributions given by (1) and (2), respectively. Confidence bounds for these two quantities are also known as tolerance bounds for the respective distributions. If a confidence bound is obtained from x_m, then (because $\exp\ (x_m)$ is a monotone function of x_m) one can convert this immediately to a bound on $t_m = \exp\ (x_m)$. Confidence bounds on u and b can likewise be converted to bounds on $\delta = \exp\ (u)$ and $\beta = 1/b$.

The problem of obtaining small-sample confidence and tolerance bounds for the two-parameter Weibull or the extreme-value distribution has been considered by several authors. Johns and Lieberman [2] generated values which, when combined with certain linear estimates of extreme-value location and scale parameters, yield lower confidence bounds on the Weibull reliability function $R\ (t_m)$ associated with a specified mission time t_m. A method of obtaining lower tolerance bounds (or lower confidence bounds

on t_γ, the reliable life associated with a specified survival proportion γ) by means of the tables given in [2] is explained in [8] .

Thoman, Bain and Antle [14] give values which may be used with maximum-likelihood estimates to obtain confidence bounds on the location parameter u and the reciprocal of the scale parameter b of the extreme-value distribution. (These, of course, can be immediately converted to confidence bounds on the two-parameter Weibull scale and shape parameters, respectively.) The same authors [15] give values for obtaining either lower tolerance bounds or lower confidence bounds on reliability $\left[R\ (t_m)\ \text{for specified}\ t_m\right]$ by use of maximum-likelihood estimates.

The values given in [2], [14] and [15] for obtaining the tolerance and confidence bounds were all generated by the use of Monte Carlo simulation procedures. The values tabulated in [11] for obtaining tolerance bounds and confidence bounds on the extreme-value scale parameter from two or three ordered observations were obtained analytically.

The two- or three-order-statistic confidence bounds associated with values tabulated in [11] have fairly high efficiencies (in terms of mean squared

error of the estimators on which they are based) with respect to the other bounds described above. The two- and three-order-statistic estimators providing the basis for these bounds do not, however, have asymptotic efficiencies of 1 with respect to Cramér-Rao bounds for regular invariant estimators (see [9]).

All the other bounds mentioned above ([2], [14] and [15]), are based on estimators with Cramér-Rao efficiencies which approach 1 as sample size n becomes large. That is, they are "asymptotically efficient". This is also true of confidence bounds based on best linear invariant estimators (estimators with smallest mean squared error among linear estimators with mean squared error invariant under translations) of u, b, and x_γ (see [7]).

Comparisons of mean squared errors given in [1] and [8] for small sample sizes indicate that the maximum-likelihood and the best linear invariant estimators are essentially equally efficient (in terms of mean squared error) for estimating b or any extreme-value percentile $x_\gamma = u + b\ \ell n\left[\ell n(1/\gamma)\right]$. Comparisons of these two estimators with other invariant estimators in [8] reveal that the other estimators tend to be less efficient for small sample sizes. The linear estimators of Johns and Lieberman [2], which are approximations to the best

linear invariant estimators, however, are shown in [8] to be very efficient relative to the maximum-likelihood and best linear invariant estimators for sample sizes of 10 or larger. The maximum-likelihood estimates are somewhat more difficult to obtain than estimates defined by either of these linear estimation procedures, since iterative procedures (usually requiring the use of a computer) are involved.

The principal advantage of the tables given in this paper over those given by Johns and Lieberman [2] and Thoman, Bain and Antle ([14] and [15]) for obtaining tolerance and/or confidence bounds is that nearly all possible type II censorings from above are considered here. That is, bounds are based on the first (smallest) m of n sample observations, $m = 3(1)n$. This allows for life-test situations in which testing is terminated at the time of the mth failure, with $3 \leq m \leq n$.

Johns and Lieberman give values for obtaining bounds on $R(t_m)$ applicable to four specified censorings for each sample of size n, n = 10, 15, 20, 30, 50, and 100. All of the confidence and tolerance bounds obtainable from tables of Thoman, Bain and Antle apply to uncensored samples of size ranging from 5 to 120 (for bounds on u and $\delta = 1/b$) and from 8 to 100 (for bounds on reliability $R(t_m)$ and reliable life t_γ. Because best linear

invariant estimator weights are available only for a limited number of sample sizes, tabulations for samples of size n, n = 3(1)25 only are given here. Tables B-1, B-2, B-3, B-4, and B-5 give values for obtaining, for the extreme-value distribution, confidence bounds on the scale parameter, b, the location parameter u (the 100 [1-exp(-1)] percent point of the distribution) and the 90th, 95th and 99th distribution percentiles, respectively. These confidence bounds can be easily converted to confidence bounds on the Weibull shape and scale parameters and Weibull tolerance bounds (or confidence bounds on t_γ) for γ = .90, .95, .99. Since percentage points of the statistics from which the bounds are obtained are tabulated for percentages of 100 times .02, .05, .10, .25, .40, .50, .60, .75, .90, .95, .98 in each case, it is possible to use the tables for testing hypotheses concerning distribution parameters as well as for obtaining confidence bounds. (That is, opposite ends of the table are used for one-sided confidence bounds and for corresponding one-sided tests.)

2. CONFIDENCE BOUNDS BASED ON BEST LINEAR INVARIANT ESTIMATORS

Coefficients of best (minimum-variance) linear unbiased estimators for the parameters u and b and for x_γ were generated by Lieblein [4] and their use investigated for complete samples of size 2 through 6. Extension to censored samples of the same size was made by Lieblein and Zelen [5]. Expressions for the first and second moments and the cross-product moments of ordered reduced extreme-value variates, which are required for computing the coefficients corresponding to the best linear unbiased estimators, had been derived previously by Lieblein [3].

We assume that a sample of size n has been randomly selected from a distribution defined by (1) and that the observations of interest consist of the natural logarithms of the n sample values. Equivalently, we may assume that the observations of interest were selected directly from the distribution defined by (2). We suppose that the sample of observations from (2) has been ordered in terms of size and may be censored at the <u>m</u>th smallest observation, $m \leq n$. The m ordered observations we denote by $X_{1,n}$, $X_{2,}$ . ... , $X_{m,n}$, with $X_{k,n} \leq X_{k+1,n}$, k = 1,2, ... , m-1. As was mentioned earlier, this corresponds to life-testing situations in which testing of items is terminated at the time of the <u>m</u>th item failure.

Suppose we consider all estimators of u, b, and x_γ which are weighted sums of $X_{1,n}, X_{2,n}, \ldots, X_{m,n}$ with mean squared error invariant under translations in X space (independent of u). The estimators among these with smallest mean squared error we call the "best linear invariant estimators". It is shown in [10] that the best linear invariant estimators $\tilde{u} \equiv \sum_{i=1}^{m} a_{i,m,n}X_{i,n}$, $\tilde{b} \equiv \sum_{i=1}^{m} c_{i,m,n}X_{i,n}$, and $\tilde{X}_\gamma \equiv \tilde{u} + \tilde{b}\, \ell n\,[\ell n(1/\gamma)]$ of u,b and x_γ, respectively, are linear functions of the best linear unbiased estimators of u and b. The coefficients $a_{i,m,n}$ and $c_{i,m,n}$ can therefore be obtained by a simple linear transformation from the coefficients of the best linear unbiased estimators. Values of $a_{i,m}$ and $c_{i,m}$ were thus calculated by first determining the coefficients of the best linear unbiased estimators using the method of Lieblein and Zelen applied to the expressions for the moments of the reduced order statistics. These values are given for m = 2(1)n, n = 2(1)15 in [7] and for m = 2(1)n, n = 2(1)25 in [6].

The estimator b, because its coefficients add to 1, is a maximal invariant which will yield confidence intervals and tests of hypotheses independent of a scale parameter u. We note that the expectation of $\tilde{b}$ is proportional to b and its variance is proportional to b^2. In fact, its $\underline{k}$th moment is proportional to b^k. Thus, the distribution of $\tilde{b}/b$ is independent of both u and b.

The coefficients of $\tilde{u}$ add to zero so that kth moments about any percentile of X are proportional to b^k. Hence $\left[\tilde{u} - \left\{u + b\, \ell n\left[\ell n\ (1/\gamma)\right]\right\}\right]/b$ has, for any γ, a distribution independent of both parameters.

We have used these invariance properties to generate, by Monte Carlo simulation procedures, random variates from the reduced extreme-value distribution and to obtain, from these, distribution percentiles of $W \equiv (\tilde{b}/b)$ and of $V_\gamma \equiv \left[\tilde{u} - \left\{u + b\, \ell n\left[\ell n\ (1/\gamma)\right]\right\}\right]/b \Big/ \left[\tilde{b}/b\right]$. Some of the details of the computations, which apply to m = 3(1)n, n = 3(1)25, are given in Section 3.

In order to obtain an upper confidence bound for b at confidence level $1-\alpha$ we note that $b \leq \tilde{b}/W_\alpha$ with probability $1-\alpha$, where W_α is the <u>100 α</u>th percentile of W, and can be found for the appropriate values of m and n in Table B-1. We reject the hypothesis H_0: $b \leq b_0$ versus H_1: $b > b_0$ at significance level α if $\tilde{b}/b_0 > W_{1-\alpha}$.

A lower confidence bound at level $1-\alpha$ on $x_\gamma \equiv u + b\, \ell n\left[\ell n\ (1/\gamma)\right]$ can be based similarly on the percentiles of V_γ given in Tables B-2, B-3, B-4, and B-5. From the definition of V_γ, we obtain $x_\gamma = \tilde{u} - \tilde{b}\, V_\gamma$. Therefore, a lower confidence bound on x_γ is given by $\tilde{u} - \tilde{b}\ (V_\gamma)_{1-\alpha}$, where $(V_\gamma)_{1-\alpha}$ is

the 100 $(1-\alpha)$th percentile of V_γ corresponding to the appropriate value of m and n, and the specified value of γ. The values in Table B-2 apply to $\gamma = 1-\exp(-1)$ so that the bounds obtainable from these values are for the parameter u. Tables B-3, B-4, and B-5 apply to x_γ for $\gamma = .90$, .95, and .99.

A $(1-\alpha)$-level confidence bound on t_γ is given by $\exp\left[\tilde{u} - \tilde{b}\,(V_\gamma)_{1-\alpha}\right]$. If $(\tilde{u} - x_m)/\tilde{b}$ is approximately equal to a value of (V_γ) tabulated, then the value of γ corresponding to V_γ represents (approximately) a confidence bound on $R(t_m)$ at confidence level $1-\alpha$. The hypothesis $X_\gamma > X_\gamma^0$ is rejected at significance level α if $(u - X_\gamma^0)/b < (V_\gamma)_\alpha$.

Before we discuss the details of the computational procedures used in generating the percentiles of W and V_γ, we remark that these tables of percentiles can be used with best linear unbiased as well as best linear invariant estimates of u and b. For u* and b*, the best linear unbiased estimators of u and b, respectively, lower confidence bounds at level $1-\alpha$ on x_γ and on b are given by

$$u^* - B\; b^*/(1 + C) - \left[b^*/(1 + C)\right] (V_\gamma)_{1-\alpha}$$

and

$$b^* / [W_\alpha (1 + C)] ,$$

where Cb^2 is the variance of b^* and Bb^2 is the covariance of u^* and b^*. This follows from linear relationships between best linear unbiased and best linear invariant estimators given in [10].

3. COMPUTATIONAL PROCEDURES AND EXAMPLE

The percentiles of W and V_γ were computed by use of Monte Carlo techniques. In general, the $\underline{100\lambda}$th percentile x_λ of an arbitrary random variate X having density $f(x)$ can be estimated by the $\underline{\lambda N}$th ordered observation $X_{(\lambda N)}$ from a sample of size N. In fact, the distribution of $X_{(\lambda N)}$ is asymptotically normal with mean x_λ and variance $\lambda(1-\lambda)/N\left[f(x_\lambda)\right]^2$ (see Mosteller [13]).

In the current situation, 20,000 samples were generated for each combination of m and n in computing the percentiles of W and each V_γ. Therefore, because of computer time and storage limitations, we constructed a histogram of each statistic rather than storing and ranking the entire 20,000 values of W and the 3 V_γ's. Each histogram was placed on the unit interval through the use of a linear transformation performed on each statistic that sent prior approximations to the first and 99th percentiles to 0.1 and 0.9, respectively. The prior approximations were made by generating 1000 samples of the W and V_γ's and saving the 10th smallest and 10th largest values. The 0.1 and 0.9 points of the unit interval were chosen as the image points of the asymptotic approximations to the first and 99th percentiles in order that any reasonable errors in these approximations would not cause loss of information in the tails of the distribution.

Each histogram consisted of 300 subintervals of equal length. The estimates of the percentiles were computed by use of linear interpolation in the histogram. The truncation error associated with this procedure is insignificant compared to the sampling error due to a finite number of Monte Carlo samples. The values tabulated are correct to within a unit in the second significant figure.

Other details applying to estimation of the percentiles from the histogram are discussed in [12], which gives percentage points of statistics computed concurrently with those considered here.

In order to illustrate the use of the tables, we refer to data shown in [7]. A complete sample of twenty-four failure times is given as:

$t_{1,24} = 6.0$	$t_{9,24} = 69.0$	$t_{17,24} = 141.0$
$t_{2,24} = 8.6$	$t_{10,24} = 74.0$	$t_{18,24} = 144.0$
$t_{3,24} = 17.8$	$t_{11,24} = 74.0$	$t_{19,24} = 146.0$
$t_{4,24} = 18.0$	$t_{12,24} = 89.0$	$t_{20,24} = 150.0$
$t_{5,24} = 27.5$	$t_{13,24} = 109.0$	$t_{21,24} = 151.0$
$t_{6,24} = 33.5$	$t_{14,24} = 118.0$	$t_{22,24} = 153.0$
$t_{7,24} = 50.5$	$t_{15,24} = 119.0$	$t_{23,24} = 153.1$
$t_{8,24} = 51.5$	$t_{16,24} = 138.0$	$t_{24,24} = 153.2$

Using logarithms $x_{i,24}$ of the failure time values $t_{i,24}$ and tables of $a_{i,24,24}$ and $c_{i,24,24}$ appearing in [8], we obtain

$$\tilde{u} = \sum_{i=1}^{24} a_{i,24,24}\, x_{i,24} = 4.614 \text{ and } \tilde{b} = \sum_{i=1}^{24} c_{i,24,24}\, x_{i,24} = 0.569.$$

Thus, a 90% lower tolerance bound for the failure time population at confidence level .90 is $\exp\left[\tilde{u} - \tilde{b}\,(3.07)\right] = \exp\,(4.614 - 1.747) = 17.64$. A 90% upper confidence bound for b is $\tilde{b}/0.77 = 0.73$.

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TABLE B-1

Percentiles of the Distribution of W

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
3	3	0.11	0.17	0.25	0.42	0.57	0.67	0.78	0.99	1.33	1.56	1.86
4	3	0.10	0.15	0.22	0.39	0.53	0.64	0.75	0.96	1.32	1.56	1.90
	4	0.20	0.28	0.37	0.54	0.68	0.77	0.86	1.05	1.33	1.53	1.77
5	3	0.09	0.14	0.21	0.37	0.51	0.61	0.73	0.94	1.32	1.59	1.93
	4	0.18	0.26	0.34	0.50	0.64	0.74	0.84	1.03	1.35	1.55	1.82
	5	0.28	0.36	0.44	0.60	0.73	0.82	0.91	1.07	1.33	1.50	1.70
6	3	0.09	0.14	0.21	0.36	0.50	0.61	0.72	0.93	1.32	1.59	1.92
	4	0.18	0.25	0.32	0.49	0.62	0.72	0.82	1.01	1.33	1.55	1.84
	5	0.25	0.33	0.42	0.58	0.71	0.79	0.89	1.05	1.33	1.51	1.73
	6	0.33	0.41	0.50	0.65	0.77	0.85	0.93	1.07	1.31	1.46	1.64
7	3	0.08	0.14	0.20	0.35	0.49	0.59	0.71	0.92	1.30	1.56	1.92
	4	0.17	0.24	0.31	0.48	0.62	0.71	0.81	1.01	1.32	1.54	1.82
	5	0.25	0.32	0.40	0.56	0.70	0.78	0.88	1.05	1.33	1.52	1.75
	6	0.32	0.39	0.47	0.63	0.75	0.84	0.92	1.07	1.32	1.48	1.67
	7	0.38	0.46	0.54	0.69	0.80	0.87	0.95	1.08	1.30	1.43	1.60
8	3	0.08	0.13	0.19	0.35	0.49	0.59	0.70	0.92	1.31	1.58	1.95
	4	0.16	0.23	0.31	0.47	0.61	0.70	0.81	1.00	1.33	1.55	1.83
	5	0.23	0.31	0.39	0.55	0.68	0.77	0.87	1.05	1.33	1.52	1.76
	6	0.30	0.38	0.46	0.62	0.74	0.82	0.91	1.06	1.32	1.49	1.69
	7	0.36	0.44	0.52	0.67	0.78	0.86	0.94	1.08	1.30	1.45	1.62
	8	0.42	0.50	0.58	0.71	0.82	0.89	0.96	1.09	1.28	1.41	1.56
9	3	0.08	0.13	0.19	0.34	0.49	0.59	0.70	0.92	1.31	1.58	1.92
	4	0.16	0.23	0.31	0.47	0.60	0.70	0.80	1.00	1.33	1.55	1.84
	5	0.23	0.31	0.39	0.54	0.68	0.77	0.86	1.04	1.33	1.52	1.76
	6	0.30	0.38	0.45	0.60	0.73	0.81	0.90	1.06	1.31	1.48	1.70
	7	0.35	0.43	0.50	0.66	0.77	0.85	0.93	1.07	1.30	1.46	1.65
	8	0.40	0.48	0.55	0.70	0.81	0.88	0.95	1.08	1.28	1.42	1.59
	9	0.45	0.53	0.60	0.74	0.84	0.90	0.97	1.08	1.27	1.39	1.53

TABLE B-1 (Continued)

Percentiles of the Distribution of W

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
10	3	0.08	0.13	0.19	0.34	0.48	0.59	0.71	0.93	1.31	1.59	1.92
	4	0.16	0.23	0.30	0.46	0.60	0.70	0.80	1.00	1.33	1.57	1.86
	5	0.23	0.30	0.38	0.54	0.68	0.77	0.86	1.04	1.33	1.53	1.77
	6	0.29	0.37	0.45	0.60	0.73	0.81	0.90	1.06	1.32	1.49	1.71
	7	0.34	0.42	0.50	0.65	0.77	0.84	0.92	1.07	1.31	1.46	1.66
	8	0.39	0.47	0.54	0.69	0.80	0.87	0.95	1.08	1.29	1.43	1.60
	9	0.43	0.51	0.59	0.73	0.83	0.89	0.96	1.08	1.28	1.40	1.55
	10	0.48	0.55	0.62	0.76	0.85	0.91	0.98	1.09	1.26	1.38	1.51
11	3	0.08	0.13	0.19	0.34	0.48	0.59	0.71	0.92	1.31	1.60	1.97
	4	0.15	0.22	0.30	0.46	0.60	0.70	0.80	1.00	1.34	1.58	1.87
	5	0.22	0.30	0.38	0.54	0.67	0.76	0.86	1.04	1.34	1.54	1.82
	6	0.28	0.36	0.44	0.60	0.73	0.81	0.90	1.07	1.33	1.52	1.73
	7	0.33	0.41	0.49	0.65	0.76	0.84	0.92	1.08	1.32	1.48	1.67
	8	0.38	0.46	0.54	0.68	0.80	0.87	0.95	1.08	1.31	1.45	1.62
	9	0.42	0.50	0.57	0.71	0.82	0.89	0.96	1.09	1.29	1.42	1.58
	10	0.46	0.54	0.61	0.74	0.85	0.91	0.98	1.09	1.27	1.38	1.53
	11	0.50	0.57	0.64	0.77	0.87	0.93	0.99	1.09	1.25	1.36	1.49
12	3	0.08	0.13	0.19	0.34	0.48	0.58	0.70	0.92	1.30	1.56	1.87
	4	0.16	0.22	0.30	0.46	0.60	0.70	0.80	1.00	1.33	1.55	1.82
	5	0.23	0.30	0.38	0.54	0.67	0.76	0.86	1.04	1.33	1.53	1.78
	6	0.29	0.36	0.44	0.60	0.72	0.81	0.90	1.06	1.33	1.49	1.72
	7	0.34	0.41	0.50	0.65	0.76	0.84	0.93	1.08	1.31	1.47	1.66
	8	0.38	0.46	0.54	0.68	0.79	0.87	0.95	1.08	1.30	1.45	1.61
	9	0.42	0.50	0.57	0.71	0.82	0.89	0.96	1.09	1.29	1.43	1.58
	10	0.45	0.53	0.61	0.74	0.84	0.90	0.97	1.09	1.28	1.40	1.55
	11	0.49	0.56	0.64	0.76	0.86	0.92	0.98	1.09	1.27	1.37	1.51
	12	0.53	0.60	0.66	0.78	0.87	0.93	0.99	1.09	1.24	1.35	1.46
13	3	0.08	0.13	0.19	0.33	0.48	0.58	0.69	0.91	1.30	1.58	1.95
	4	0.15	0.22	0.29	0.45	0.59	0.69	0.79	0.99	1.33	1.57	1.86
	5	0.22	0.30	0.37	0.53	0.67	0.75	0.85	1.03	1.34	1.55	1.79
	6	0.28	0.36	0.43	0.59	0.72	0.80	0.89	1.06	1.33	1.51	1.72
	7	0.33	0.40	0.48	0.64	0.76	0.84	0.92	1.07	1.32	1.48	1.67
	8	0.37	0.45	0.53	0.67	0.79	0.86	0.94	1.08	1.30	1.45	1.62
	9	0.42	0.49	0.56	0.70	0.81	0.88	0.95	1.08	1.29	1.42	1.58
	10	0.44	0.52	0.60	0.73	0.83	0.89	0.96	1.08	1.28	1.40	1.55
	11	0.48	0.55	0.62	0.75	0.85	0.91	0.97	1.08	1.26	1.38	1.51
	12	0.51	0.58	0.65	0.77	0.86	0.92	0.98	1.08	1.25	1.36	1.47
	13	0.54	0.61	0.68	0.79	0.88	0.93	0.99	1.09	1.24	1.33	1.44

TABLE B-1-(Continued)

Percentiles of the Distribution of W

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
14	3	0.08	0.13	0.19	0.34	0.48	0.58	0.69	0.91	1.31	1.58	1.94
	4	0.16	0.22	0.30	0.45	0.59	0.69	0.80	0.99	1.33	1.57	1.86
	5	0.22	0.30	0.37	0.53	0.67	0.76	0.86	1.04	1.34	1.54	1.77
	6	0.28	0.35	0.43	0.59	0.72	0.81	0.90	1.06	1.33	1.51	1.71
	7	0.33	0.40	0.48	0.64	0.76	0.84	0.93	1.07	1.32	1.48	1.67
	8	0.38	0.45	0.53	0.67	0.79	0.86	0.94	1.08	1.30	1.45	1.63
	9	0.41	0.49	0.56	0.70	0.82	0.88	0.96	1.08	1.29	1.42	1.59
	10	0.45	0.52	0.59	0.72	0.83	0.90	0.96	1.09	1.28	1.40	1.55
	11	0.48	0.55	0.62	0.75	0.85	0.91	0.97	1.09	1.26	1.38	1.52
	12	0.50	0.57	0.64	0.77	0.86	0.92	0.98	1.09	1.25	1.36	1.49
	13	0.53	0.60	0.67	0.79	0.88	0.93	0.99	1.09	1.24	1.34	1.46
	14	0.57	0.63	0.69	0.81	0.89	0.94	0.99	1.09	1.23	1.32	1.43
15	3	0.08	0.13	0.19	0.33	0.47	0.57	0.68	0.90	1.29	1.57	1.92
	4	0.16	0.22	0.29	0.45	0.59	0.68	0.79	0.99	1.33	1.56	1.85
	5	0.22	0.29	0.37	0.53	0.66	0.75	0.85	1.04	1.33	1.53	1.79
	6	0.28	0.35	0.44	0.59	0.71	0.80	0.89	1.06	1.32	1.50	1.71
	7	0.33	0.41	0.49	0.63	0.75	0.83	0.92	1.07	1.32	1.48	1.67
	8	0.37	0.45	0.52	0.67	0.78	0.86	0.94	1.08	1.30	1.45	1.63
	9	0.41	0.49	0.56	0.69	0.81	0.88	0.95	1.08	1.29	1.43	1.59
	10	0.45	0.52	0.59	0.72	0.82	0.89	0.96	1.09	1.28	1.41	1.56
	11	0.48	0.54	0.61	0.74	0.84	0.90	0.97	1.09	1.27	1.39	1.54
	12	0.50	0.57	0.63	0.76	0.86	0.91	0.98	1.09	1.26	1.37	1.50
	13	0.52	0.59	0.66	0.78	0.87	0.92	0.98	1.08	1.25	1.35	1.47
	14	0.55	0.62	0.68	0.80	0.88	0.94	0.99	1.08	1.24	1.33	1.45
	15	0.58	0.64	0.70	0.81	0.89	0.94	0.99	1.09	1.23	1.32	1.42
16	3	0.08	0.13	0.19	0.33	0.47	0.56	0.68	0.90	1.29	1.58	1.94
	4	0.15	0.22	0.29	0.45	0.59	0.68	0.79	0.99	1.33	1.56	1.86
	5	0.22	0.29	0.36	0.53	0.66	0.75	0.85	1.03	1.33	1.54	1.78
	6	0.27	0.35	0.43	0.58	0.71	0.79	0.88	1.05	1.33	1.51	1.74
	7	0.31	0.40	0.48	0.63	0.75	0.83	0.91	1.07	1.31	1.47	1.69
	8	0.36	0.44	0.52	0.66	0.78	0.85	0.93	1.07	1.30	1.45	1.64
	9	0.40	0.48	0.55	0.69	0.80	0.87	0.94	1.08	1.29	1.43	1.60
	10	0.43	0.51	0.58	0.72	0.82	0.89	0.96	1.08	1.28	1.41	1.57
	11	0.46	0.53	0.61	0.74	0.84	0.90	0.97	1.09	1.27	1.39	1.54
	12	0.49	0.56	0.63	0.76	0.85	0.91	0.97	1.09	1.26	1.38	1.50
	13	0.51	0.59	0.65	0.77	0.86	0.92	0.98	1.09	1.25	1.36	1.48
	14	0.54	0.61	0.67	0.79	0.88	0.93	0.99	1.09	1.24	1.34	1.46
	15	0.56	0.63	0.69	0.80	0.89	0.94	0.99	1.09	1.23	1.32	1.43
	16	0.59	0.65	0.71	0.82	0.90	0.95	1.00	1.09	1.21	1.29	1.40

TABLE B-1 - (Continued)

Percentiles of the Distribution of W

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
17	3	0.08	0.13	0.18	0.33	0.48	0.58	0.69	0.92	1.33	1.59	1.95
	4	0.15	0.22	0.30	0.45	0.59	0.69	0.80	1.00	1.35	1.58	1.87
	5	0.22	0.30	0.37	0.53	0.67	0.76	0.86	1.04	1.34	1.55	1.79
	6	0.28	0.35	0.43	0.59	0.71	0.80	0.89	1.06	1.33	1.52	1.73
	7	0.33	0.40	0.48	0.63	0.75	0.83	0.92	1.07	1.32	1.48	1.68
	8	0.37	0.44	0.52	0.67	0.78	0.86	0.94	1.08	1.31	1.47	1.63
	9	0.40	0.48	0.55	0.70	0.80	0.88	0.95	1.09	1.30	1.44	1.60
	10	0.44	0.51	0.58	0.72	0.82	0.89	0.96	1.09	1.29	1.42	1.58
	11	0.46	0.54	0.61	0.74	0.84	0.90	0.97	1.09	1.28	1.39	1.55
	12	0.49	0.56	0.63	0.76	0.85	0.91	0.98	1.09	1.27	1.38	1.50
	13	0.51	0.58	0.65	0.78	0.87	0.92	0.99	1.09	1.26	1.36	1.48
	14	0.53	0.61	0.67	0.79	0.88	0.93	0.99	1.09	1.24	1.34	1.46
	15	0.56	0.63	0.69	0.80	0.89	0.94	1.00	1.09	1.23	1.33	1.44
	16	0.58	0.65	0.71	0.82	0.90	0.95	1.00	1.09	1.22	1.31	1.41
	17	0.61	0.67	0.73	0.83	0.91	0.95	1.00	1.09	1.21	1.29	1.39
18	3	0.07	0.12	0.19	0.33	0.48	0.58	0.69	0.92	1.31	1.59	1.93
	4	0.15	0.22	0.29	0.45	0.59	0.69	0.79	0.99	1.34	1.58	1.87
	5	0.22	0.29	0.37	0.53	0.66	0.75	0.85	1.03	1.34	1.54	1.79
	6	0.27	0.35	0.43	0.58	0.71	0.80	0.89	1.05	1.32	1.51	1.73
	7	0.33	0.40	0.47	0.62	0.75	0.83	0.91	1.07	1.31	1.48	1.68
	8	0.37	0.44	0.51	0.66	0.77	0.85	0.93	1.08	1.30	1.46	1.63
	9	0.40	0.47	0.55	0.69	0.80	0.87	0.95	1.08	1.30	1.43	1.59
	10	0.43	0.51	0.58	0.71	0.81	0.88	0.96	1.08	1.28	1.41	1.56
	11	0.46	0.53	0.60	0.73	0.83	0.90	0.97	1.08	1.27	1.39	1.54
	12	0.49	0.56	0.63	0.75	0.85	0.91	0.97	1.09	1.26	1.38	1.52
	13	0.51	0.58	0.65	0.77	0.86	0.92	0.98	1.09	1.25	1.36	1.48
	14	0.54	0.60	0.66	0.78	0.87	0.93	0.99	1.09	1.25	1.35	1.46
	15	0.56	0.62	0.68	0.79	0.88	0.93	0.99	1.08	1.23	1.33	1.44
	16	0.57	0.64	0.70	0.81	0.89	0.94	0.99	1.09	1.22	1.31	1.42
	17	0.59	0.65	0.71	0.82	0.90	0.95	1.00	1.09	1.22	1.30	1.39
	18	0.61	0.67	0.73	0.83	0.91	0.95	1.00	1.08	1.20	1.28	1.37
19	3	0.08	0.12	0.18	0.33	0.47	0.57	0.69	0.91	1.30	1.57	1.91
	4	0.15	0.22	0.29	0.45	0.59	0.68	0.79	0.99	1.33	1.56	1.84
	5	0.22	0.29	0.37	0.52	[illegible]	0.76	0.85	1.03	1.33	1.54	1.78
	6	0.28	0.35	0.43	0.58	0.71	0.80	0.89	1.05	1.33	1.51	1.74
	7	0.32	0.40	0.48	0.63	0.75	[illegible]	0.91	1.06	1.32	1.48	1.69
	8	0.36	0.44	0.51	0.66	0.78	0.85	0.93	1.08	1.31	1.46	1.65
	9	0.40	0.47	0.55	0.69	0.80	0.87	0.95	1.08	1.30	1.43	1.60
	10	0.43	0.50	0.57	0.71	0.82	0.89	0.96	1.09	1.28	1.42	1.57
	11	0.46	0.53	0.60	0.73	0.83	0.90	0.97	1.09	1.27	1.40	1.55
	12	0.48	0.55	0.62	0.75	0.85	0.91	0.97	1.09	1.26	1.38	1.51
	13	0.50	0.58	0.64	0.77	0.86	0.92	0.98	1.09	1.26	1.37	1.49
	14	0.53	0.60	0.67	0.78	0.87	0.93	0.98	1.09	1.25	1.35	1.47
	15	0.54	0.62	0.68	0.79	0.88	0.93	0.99	1.09	1.24	1.34	1.45
	16	0.56	0.63	0.69	0.81	0.89	0.94	0.99	1.09	1.23	1.32	1.42
	17	0.58	0.65	0.71	0.82	0.90	0.95	1.00	1.09	1.22	1.31	1.41
	18	0.60	0.67	0.73	0.83	0.90	0.95	1.00	1.08	1.21	1.29	1.39
	19	0.62	0.68	0.74	0.84	0.91	0.96	1.00	1.08	1.20	1.28	1.36

TABLE B-1 - (Continued)

Percentiles of the Distribution of W

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
20	3	0.07	0.12	0.18	0.33	0.47	0.58	0.69	0.91	1.31	1.60	1.97
	4	0.15	0.22	0.29	0.45	0.59	0.69	0.79	0.99	1.34	1.57	1.89
	5	0.22	0.29	0.37	0.52	0.66	0.75	0.85	1.03	1.34	1.55	1.81
	6	0.27	0.35	0.43	0.58	0.71	0.79	0.89	1.05	1.33	1.52	1.75
	7	0.32	0.40	0.47	0.62	0.75	0.83	0.91	1.07	1.32	1.49	1.70
	8	0.35	0.43	0.51	0.66	0.78	0.85	0.93	1.08	1.30	1.46	1.65
	9	0.39	0.47	0.55	0.69	0.80	0.87	0.95	1.08	1.30	1.44	1.61
	10	0.43	0.50	0.57	0.71	0.82	0.89	0.96	1.09	1.29	1.42	1.58
	11	0.45	0.53	0.60	0.73	0.83	0.90	0.97	1.09	1.28	1.40	1.54
	12	0.48	0.55	0.62	0.75	0.85	0.91	0.98	1.09	1.27	1.38	1.52
	13	0.50	0.57	0.64	0.77	0.86	0.92	0.98	1.09	1.26	1.36	1.50
	14	0.52	0.60	0.66	0.78	0.87	0.93	0.99	1.09	1.25	1.35	1.48
	15	0.54	0.61	0.68	0.79	0.88	0.93	0.99	1.09	1.24	1.34	1.46
	16	0.56	0.63	0.69	0.81	0.89	0.94	1.00	1.09	1.23	1.33	1.44
	17	0.58	0.65	0.71	0.82	0.90	0.95	1.00	1.09	1.22	1.31	1.42
	18	0.60	0.66	0.72	0.83	0.90	0.95	1.00	1.08	1.22	1.30	1.40
	19	0.62	0.68	0.74	0.84	0.91	0.96	1.00	1.08	1.21	1.28	1.37
	20	0.64	0.70	0.75	0.85	0.92	0.96	1.01	1.08	1.20	1.27	1.36
21	3	0.07	0.12	0.18	0.33	0.47	0.57	0.69	0.91	1.30	1.58	1.90
	4	0.15	0.22	0.29	0.45	0.59	0.68	0.79	0.99	1.33	1.57	1.87
	5	0.22	0.29	0.36	0.53	0.66	0.75	0.85	1.03	1.34	1.55	1.80
	6	0.28	0.35	0.43	0.58	0.71	0.80	0.89	1.06	1.34	1.52	1.73
	7	0.32	0.40	0.47	0.62	0.74	0.82	0.91	1.07	1.33	1.49	1.69
	8	0.36	0.43	0.51	0.66	0.77	0.85	0.93	1.08	1.32	1.47	1.66
	9	0.40	0.47	0.54	0.68	0.80	0.87	0.95	1.08	1.30	1.44	1.62
	10	0.42	0.50	0.57	0.71	0.82	0.88	0.96	1.09	1.29	1.42	1.58
	11	0.45	0.52	0.60	0.73	0.83	0.90	0.96	1.08	1.28	1.40	1.55
	12	0.47	0.55	0.61	0.75	0.84	0.91	0.97	1.09	1.27	1.39	1.53
	13	0.50	0.57	0.64	0.76	0.86	0.92	0.98	1.09	1.26	1.37	1.50
	14	0.52	0.59	0.65	0.77	0.86	0.92	0.98	1.09	1.25	1.36	1.48
	15	0.54	0.50	0.67	0.79	0.88	0.93	0.99	1.09	1.25	1.34	1.46
	16	0.56	0.63	0.68	0.80	0.88	0.94	0.99	1.09	1.24	1.33	1.44
	17	0.57	0.64	0.70	0.81	0.89	0.94	1.00	1.09	1.23	1.32	1.42
	18	0.59	0.66	0.71	0.82	0.90	0.95	1.00	1.09	1.22	1.30	1.41
	19	0.61	0.67	0.73	0.83	0.91	0.95	1.00	1.09	1.21	1.29	1.39
	20	0.62	0.68	0.74	0.84	0.91	0.96	1.00	1.09	1.21	1.28	1.37
	21	0.64	0.70	0.76	0.85	0.92	0.96	1.01	1.08	1.19	1.26	1.34

TABLE B-1 - (Continued)

Percentiles of the Distribution of W

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
22	3	0.07	0.12	0.18	0.33	0.47	0.57	0.69	0.92	1.32	1.59	1.93
	4	0.15	0.22	0.29	0.45	0.59	0.68	0.79	0.99	1.33	1.57	1.88
	5	0.22	0.29	0.37	0.53	0.66	0.75	0.85	1.04	1.34	1.54	1.81
	6	0.27	0.35	0.42	0.58	0.71	0.80	0.89	1.06	1.33	1.53	1.74
	7	0.32	0.39	0.47	0.62	0.75	0.83	0.92	1.07	1.33	1.49	1.69
	8	0.36	0.43	0.51	0.66	0.77	0.85	0.93	1.08	1.31	1.47	1.65
	9	0.39	0.47	0.54	0.69	0.80	0.87	0.95	1.09	1.30	1.44	1.61
	10	0.43	0.50	0.57	0.71	0.81	0.89	0.96	1.09	1.29	1.42	1.58
	11	0.45	0.52	0.60	0.73	0.83	0.90	0.97	1.09	1.28	1.41	1.55
	12	0.48	0.54	0.62	0.74	0.84	0.91	0.98	1.09	1.27	1.38	1.52
	13	0.50	0.57	0.64	0.76	0.85	0.92	0.98	1.09	1.26	1.37	1.49
	14	0.52	0.59	0.66	0.77	0.87	0.92	0.99	1.09	1.25	1.35	1.46
	15	0.53	0.61	0.67	0.79	0.88	0.93	0.99	1.09	1.24	1.34	1.46
	16	0.56	0.62	0.69	0.80	0.88	0.94	0.99	1.09	1.24	1.33	1.43
	17	0.57	0.64	0.70	0.81	0.89	0.94	1.00	1.09	1.23	1.32	1.42
	18	0.59	0.65	0.71	0.82	0.90	0.95	1.00	1.09	1.22	1.31	1.40
	19	0.61	0.67	0.72	0.83	0.90	0.95	1.00	1.08	1.21	1.29	1.39
	20	0.62	0.68	0.74	0.84	0.91	0.95	1.00	1.08	1.21	1.28	1.37
	21	0.64	0.70	0.75	0.85	0.91	0.96	1.00	1.08	1.20	1.27	1.36
	22	0.65	0.71	0.76	0.85	0.92	0.96	1.01	1.08	1.19	1.26	1.33
23	3	0.07	0.12	0.18	0.32	0.46	0.56	0.68	0.91	1.31	1.59	1.94
	4	0.14	0.21	0.28	0.44	0.58	0.68	0.79	0.99	1.33	1.57	1.86
	5	0.21	0.28	0.35	0.52	0.65	0.74	0.85	1.03	1.33	1.55	1.79
	6	0.27	0.34	0.42	0.57	0.70	0.79	0.88	1.05	1.34	1.51	1.73
	7	0.31	0.39	0.47	0.62	0.75	0.83	0.91	1.07	1.32	1.48	1.68
	8	0.35	0.43	0.51	0.66	0.77	0.85	0.93	1.08	1.31	1.47	1.65
	9	0.39	0.47	0.54	0.69	0.80	0.87	0.95	1.08	1.30	1.45	1.61
	10	0.42	0.50	0.57	0.71	0.82	0.88	0.95	1.09	1.29	1.43	1.59
	11	0.45	0.52	0.59	0.73	0.83	0.89	0.96	1.09	1.28	1.40	1.57
	12	0.47	0.55	0.62	0.75	0.84	0.91	0.97	1.09	1.27	1.38	1.54
	13	0.50	0.57	0.63	0.76	0.85	0.91	0.98	1.09	1.26	1.37	1.52
	14	0.52	0.58	0.65	0.78	0.86	0.92	0.98	1.09	1.25	1.36	1.48
	15	0.53	0.60	0.67	0.79	0.87	0.93	0.99	1.09	1.25	1.35	1.47
	16	0.55	0.62	0.68	0.80	0.88	0.93	0.99	1.09	1.24	1.33	1.45
	17	0.57	0.64	0.70	0.81	0.89	0.94	0.99	1.09	1.23	1.33	1.43
	18	0.59	0.65	0.71	0.82	0.90	0.94	1.00	1.09	1.22	1.31	1.42
	19	0.60	0.66	0.72	0.83	0.90	0.95	1.00	1.08	1.22	1.30	1.40
	20	0.62	0.68	0.73	0.83	0.91	0.95	1.00	1.09	1.21	1.29	1.38
	21	0.63	0.69	0.74	0.84	0.91	0.96	1.00	1.08	1.20	1.28	1.37
	22	0.65	0.70	0.76	0.85	0.92	0.96	1.01	1.08	1.19	1.27	1.35
	23	0.66	0.72	0.77	0.86	0.92	0.96	1.01	1.08	1.19	1.26	1.34

TABLE B-1 - (Concluded)

Percentiles of the Distribution of W

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
24	3	0.07	0.12	0.18	0.33	0.47	0.57	0.69	0.92	1.29	1.56	1.90
	4	0.15	0.21	0.28	0.44	0.59	0.69	0.79	0.99	1.32	1.56	1.85
	5	0.21	0.28	0.36	0.52	0.65	0.75	0.85	1.03	1.33	1.54	1.79
	6	0.27	0.34	0.42	0.57	0.70	0.79	0.89	1.05	1.33	1.51	1.73
	7	0.32	0.39	0.47	0.62	0.74	0.82	0.91	1.07	1.32	1.49	1.69
	8	0.35	0.43	0.50	0.65	0.77	0.85	0.93	1.07	1.31	1.46	1.65
	9	0.39	0.47	0.54	0.68	0.79	0.86	0.94	1.08	1.30	1.44	1.61
	10	0.42	0.50	0.56	0.70	0.81	0.88	0.95	1.09	1.29	1.42	1.58
	11	0.45	0.52	0.59	0.72	0.83	0.89	0.96	1.09	1.28	1.41	1.56
	12	0.47	0.54	0.61	0.74	0.84	0.90	0.97	1.09	1.27	1.39	1.54
	13	0.49	0.56	0.63	0.76	0.85	0.91	0.98	1.09	1.27	1.38	1.52
	14	0.52	0.58	0.65	0.77	0.86	0.92	0.98	1.09	1.26	1.36	1.49
	15	0.54	0.60	0.67	0.78	0.87	0.92	0.98	1.09	1.25	1.35	1.47
	16	0.55	0.62	0.68	0.79	0.88	0.93	0.99	1.08	1.24	1.34	1.45
	17	0.57	0.63	0.69	0.80	0.88	0.94	0.99	1.08	1.24	1.33	1.44
	18	0.58	0.65	0.70	0.81	0.89	0.94	0.99	1.08	1.23	1.32	1.43
	19	0.60	0.66	0.72	0.82	0.90	0.95	1.00	1.08	1.22	1.31	1.40
	20	0.62	0.67	0.73	0.83	0.90	0.95	1.00	1.08	1.21	1.29	1.39
	21	0.63	0.68	0.74	0.84	0.91	0.95	1.00	1.08	1.21	1.28	1.37
	22	0.64	0.69	0.75	0.84	0.91	0.96	1.00	1.08	1.20	1.27	1.36
	23	0.65	0.71	0.76	0.85	0.92	0.96	1.00	1.08	1.19	1.26	1.34
	24	0.66	0.72	0.77	0.86	0.92	0.96	1.01	1.08	1.19	1.25	1.33
25	3	0.07	0.12	0.18	0.33	0.47	0.57	0.68	0.90	1.29	1.57	1.91
	4	0.15	0.21	0.28	0.45	0.59	0.68	0.79	0.99	1.32	1.56	1.85
	5	0.21	0.28	0.37	0.52	0.66	0.74	0.84	1.02	1.32	1.54	1.80
	6	0.27	0.34	0.42	0.58	0.70	0.79	0.88	1.04	1.31	1.50	1.74
	7	0.32	0.39	0.47	0.62	0.74	0.82	0.90	1.06	1.31	1.48	1.69
	8	0.35	0.43	0.51	0.65	0.77	0.84	0.92	1.07	1.30	1.45	1.65
	9	0.39	0.46	0.54	0.68	0.79	0.86	0.94	1.07	1.29	1.44	1.62
	10	0.42	0.49	0.57	0.70	0.81	0.88	0.95	1.08	1.28	1.42	1.59
	11	0.44	0.52	0.59	0.72	0.82	0.89	0.96	1.08	1.28	1.41	1.57
	12	0.47	0.54	0.61	0.74	0.84	0.90	0.97	1.08	1.27	1.39	1.54
	13	0.49	0.56	0.63	0.76	0.85	0.91	0.98	1.09	1.26	1.37	1.51
	14	0.51	0.58	0.65	0.77	0.86	0.92	0.98	1.09	1.26	1.36	1.49
	15	0.53	0.60	0.66	0.78	0.87	0.92	0.98	1.09	1.25	1.35	1.47
	16	0.55	0.61	0.68	0.79	0.88	0.93	0.99	1.08	1.24	1.33	1.45
	17	0.56	0.63	0.69	0.80	0.88	0.94	0.99	1.09	1.23	1.32	1.44
	18	0.58	0.64	0.70	0.81	0.89	0.94	0.99	1.09	1.22	1.31	1.42
	19	0.59	0.66	0.71	0.82	0.90	0.95	1.00	1.08	1.22	1.30	1.41
	20	0.61	0.67	0.73	0.83	0.90	0.95	1.00	1.08	1.21	1.30	1.39
	21	0.62	0.68	0.74	0.83	0.91	0.95	1.00	1.08	1.21	1.29	1.38
	22	0.64	0.69	0.75	0.84	0.91	0.96	1.00	1.08	1.20	1.28	1.36
	23	0.65	0.70	0.76	0.85	0.92	0.96	1.00	1.08	1.19	1.27	1.35
	24	0.66	0.72	0.77	0.86	0.92	0.96	1.00	1.08	1.19	1.26	1.34
	25	0.67	0.73	0.78	0.86	0.93	0.97	1.01	1.08	1.18	1.25	1.32

TABLE B-2

Percentiles of the Distribution of $(\tilde{u}-u)/\tilde{b}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
3	3	-4.47	-2.54	-1.49	-0.52	-0.10	0.10	0.31	0.69	1.46	2.12	3.39
4	3	-6.92	-3.85	-2.32	-0.84	-0.29	-0.04	0.18	0.50	1.06	1.55	2.43
	4	-2.37	-1.50	-0.96	-0.37	-0.08	0.09	0.25	0.55	1.07	1.49	2.15
5	3	-9.35	-5.22	-3.04	-1.22	-0.50	-0.19	0.06	0.40	0.86	1.20	1.76
	4	-3.13	-1.94	-1.24	-0.50	-0.16	0.02	0.18	0.45	0.88	1.22	1.74
	5	-1.63	-1.08	-0.73	-0.31	-0.06	0.08	0.22	0.47	0.89	1.20	1.64
6	3	-10.54	-6.12	-3.72	-1.56	-0.69	-0.32	-0.04	0.33	0.75	1.02	1.39
	4	-3.69	-2.39	-1.59	-0.67	-0.25	-0.05	0.12	0.38	0.76	1.03	1.42
	5	-2.05	-1.36	-0.91	-0.38	-0.11	0.04	0.17	0.40	0.77	1.04	1.41
	6	-1.29	-0.91	-0.64	-0.28	-0.06	0.07	0.19	0.41	0.77	1.04	1.39
7	3	-13.00	-7.39	-4.45	-1.87	-0.89	-0.48	-0.16	0.26	0.68	0.90	1.20
	4	-4.67	-2.95	-1.94	-0.84	-0.36	-0.13	0.05	0.32	0.66	0.89	1.20
	5	-2.48	-1.59	-1.10	-0.48	-0.17	-0.02	0.12	0.34	0.66	0.89	1.21
	6	-1.54	-1.04	-0.73	-0.32	-0.10	0.03	0.15	0.35	0.67	0.90	1.20
	7	-1.09	-0.79	-0.56	-0.26	-0.06	0.05	0.17	0.36	0.68	0.90	1.18
8	3	-14.36	-8.15	-5.01	-2.14	-1.04	-0.58	-0.21	0.24	0.67	0.88	1.12
	4	-5.34	-3.30	-2.18	-0.99	-0.43	-0.19	0.02	0.30	0.64	0.83	1.07
	5	-2.78	-1.86	-1.25	-0.56	-0.22	-0.05	0.10	0.32	0.62	0.82	1.07
	6	-1.80	-1.20	-0.83	-0.36	-0.17	0.01	0.13	0.33	0.63	0.82	1.08
	7	-1.28	-0.88	-0.61	-0.27	-0.07	0.04	0.15	0.33	0.63	0.82	1.08
	8	-0.97	-0.70	-0.50	-0.22	-0.05	0.06	0.16	0.34	0.63	0.82	1.07
9	3	-15.68	-9.12	-5.64	-2.38	-1.17	-0.66	-0.28	0.20	0.66	0.86	1.06
	4	-6.31	-3.78	-2.47	-1.08	-0.50	-0.24	-0.01	0.28	0.61	0.79	1.00
	5	-3.19	-2.10	-1.40	-0.63	-0.26	-0.08	0.08	0.30	0.58	0.76	0.98
	6	-2.01	-1.38	-0.96	-0.41	-0.15	-0.01	0.11	0.30	0.57	0.76	0.99
	7	-1.43	-0.99	-0.70	-0.31	-0.10	0.02	0.13	0.31	0.57	0.76	0.99
	8	-1.08	-0.76	-0.55	-0.25	-0.07	0.04	0.14	0.31	0.58	0.76	0.99
	9	-0.87	-0.64	-0.47	-0.21	-0.05	0.05	0.15	0.32	0.58	0.76	0.98

TABLE B-2 - (Continued)

Percentiles of the Distribution of $(\tilde{u}-u)/\tilde{b}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
10	3	-17.45	-9.98	-6.05	-2.58	-1.29	-0.76	-0.34	0.17	0.66	0.87	1.07
	4	-6.54	-4.17	-2.70	-1.22	-0.58	-0.28	-0.04	0.27	0.60	0.77	0.96
	5	-3.56	-2.37	-1.56	-0.73	-0.31	-0.12	0.05	0.28	0.56	0.72	0.93
	6	-2.21	-1.51	-1.03	-0.48	-0.19	-0.04	0.09	0.28	0.54	0.71	0.92
	7	-1.56	-1.08	-0.77	-0.35	-0.12	-0.00	0.11	0.28	0.54	0.70	0.93
	8	-1.20	-0.86	-0.62	-0.27	-0.08	0.02	0.12	0.28	0.53	0.71	0.93
	9	-0.97	-0.70	-0.50	-0.23	-0.06	0.04	0.13	0.29	0.54	0.71	0.93
	10	-0.80	-0.60	-0.44	-0.20	-0.04	0.04	0.14	0.29	0.54	0.71	0.92
11	3	-18.52	-10.68	-6.42	-2.76	-1.41	-0.85	-0.42	0.13	0.65	0.87	1.07
	4	-7.26	-4.57	-2.95	-1.37	-0.66	-0.36	-0.10	0.24	0.58	0.75	0.92
	5	-4.00	-2.58	-1.75	-0.81	-0.37	-0.16	0.01	0.26	0.54	0.69	0.88
	6	-2.45	-1.67	-1.16	-0.53	-0.22	-0.07	0.06	0.26	0.52	0.66	0.85
	7	-1.70	-1.21	-0.85	-0.40	-0.15	-0.02	0.09	0.26	0.50	0.65	0.86
	8	-1.30	-0.92	-0.66	-0.30	-0.11	0.00	0.10	0.26	0.50	0.65	0.86
	9	-1.06	-0.76	-0.54	-0.25	-0.08	0.02	0.11	0.26	0.50	0.65	0.86
	10	-0.87	-0.63	-0.46	-0.21	-0.06	0.03	0.12	0.27	0.50	0.65	0.86
	11	-0.75	-0.55	-0.42	-0.19	-0.05	0.03	0.12	0.27	0.50	0.65	0.85
12	3	-19.08	-11.23	-6.92	-3.03	-1.58	-0.97	-0.49	0.10	0.64	0.88	1.10
	4	-7.44	-4.81	-3.17	-1.47	-0.74	-0.40	-0.14	0.21	0.58	0.75	0.92
	5	-4.17	-2.72	-1.88	-0.89	-0.42	-0.20	-0.01	0.24	0.53	0.68	0.84
	6	-2.63	-1.83	-1.27	-0.60	-0.26	-0.10	0.05	0.25	0.50	0.64	0.81
	7	-1.91	-1.32	-0.92	-0.42	-0.17	-0.04	0.08	0.25	0.48	0.62	0.80
	8	-1.41	-1.00	-0.71	-0.33	-0.12	-0.01	0.09	0.25	0.48	0.62	0.79
	9	-1.15	-0.80	-0.58	-0.27	-0.09	0.01	0.10	0.25	0.47	0.62	0.80
	10	-0.91	-0.67	-0.48	-0.23	-0.07	0.02	0.11	0.25	0.47	0.62	0.80
	11	-0.78	-0.58	-0.43	-0.20	-0.06	0.03	0.11	0.25	0.47	0.62	0.80
	12	-0.69	-0.53	-0.39	-0.19	-0.05	0.03	0.11	0.25	0.47	0.62	0.79
13	3	-19.77	-11.66	-7.41	-3.21	-1.64	-1.02	-0.54	0.08	0.65	0.88	1.09
	4	-8.72	-5.21	-3.37	-1.60	-0.82	-0.48	-0.19	0.20	0.59	0.76	0.93
	5	-4.44	-2.95	-1.99	-0.96	-0.47	-0.24	-0.04	0.24	0.54	0.68	0.84
	6	-2.86	-1.94	-1.35	-0.66	-0.31	-0.13	0.03	0.25	0.51	0.64	0.79
	7	-2.04	-1.40	-0.98	-0.46	-0.19	-0.06	0.06	0.25	0.47	0.61	0.77
	8	-1.52	-1.06	-0.77	-0.36	-0.14	-0.02	0.08	0.24	0.46	0.59	0.75
	9	-1.18	-0.86	-0.61	-0.29	-0.10	-0.00	0.09	0.24	0.45	0.58	0.74
	10	-1.00	-0.72	-0.52	-0.24	-0.08	0.01	0.10	0.24	0.45	0.58	0.74
	11	-0.85	-0.63	-0.45	-0.21	-0.06	0.02	0.11	0.24	0.45	0.58	0.75
	12	-0.74	-0.56	-0.41	-0.19	-0.05	0.03	0.11	0.25	0.45	0.59	0.75
	13	-0.67	-0.51	-0.38	-0.18	-0.05	0.04	0.11	0.25	0.45	0.59	0.75

TABLE B-2 - (Continued)

Percentiles of the Distribution of $(\tilde{u}-u)/\tilde{b}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
14	3	-21.43	-12.49	-7.65	-3.31	-1.71	-1.08	-0.57	0.06	0.65	0.90	1.11
	4	-8.30	-5.38	-3.53	-1.68	-0.87	-0.49	-0.20	0.19	0.59	0.77	0.96
	5	-4.72	-3.13	-2.17	-1.03	-0.51	-0.26	-0.04	0.24	0.54	0.69	0.84
	6	-3.07	-2.10	-1.45	-0.70	-0.32	-0.14	0.02	0.24	0.50	0.63	0.79
	7	-2.16	-1.50	-1.06	-0.50	-0.22	-0.07	0.06	0.24	0.47	0.60	0.75
	8	-1.67	-1.15	-0.81	-0.39	-0.15	-0.04	0.08	0.24	0.45	0.58	0.73
	9	-1.30	-0.93	-0.66	-0.30	-0.11	-0.01	0.09	0.23	0.44	0.56	0.72
	10	-1.07	-0.76	-0.54	-0.26	-0.09	0.00	0.09	0.23	0.43	0.56	0.72
	11	-0.89	-0.65	-0.48	-0.22	-0.07	0.01	0.09	0.23	0.43	0.56	0.72
	12	-0.76	-0.57	-0.42	-0.19	-0.06	0.02	0.10	0.23	0.43	0.56	0.72
	13	-0.68	-0.51	-0.38	-0.18	-0.05	0.02	0.10	0.23	0.43	0.56	0.72
	14	-0.63	-0.47	-0.36	-0.17	-0.05	0.03	0.10	0.23	0.43	0.56	0.72
15	3	-23.14	-13.14	-8.14	-3.63	-1.92	-1.20	-0.65	0.02	0.64	0.89	1.12
	4	-8.79	-5.55	-3.74	-1.78	-0.94	-0.55	-0.23	0.19	0.60	0.78	0.95
	5	-4.88	-3.35	-2.27	-1.10	-0.56	-0.29	-0.07	0.23	0.55	0.70	0.85
	6	-3.21	-2.21	-1.55	-0.75	-0.36	-0.17	-0.00	0.23	0.50	0.64	0.78
	7	-2.29	-1.56	-1.11	-0.55	-0.25	-0.09	0.04	0.23	0.47	0.59	0.74
	8	-1.72	-1.20	-0.86	-0.42	-0.18	-0.05	0.06	0.23	0.45	0.57	0.71
	9	-1.35	-0.96	-0.70	-0.35	-0.13	-0.03	0.07	0.23	0.43	0.56	0.69
	10	-1.10	-0.82	-0.59	-0.28	-0.10	-0.01	0.08	0.23	0.42	0.55	0.68
	11	-0.96	-0.70	-0.51	-0.24	-0.08	0.01	0.09	0.23	0.42	0.54	0.69
	12	-0.83	-0.62	-0.45	-0.21	-0.07	0.01	0.09	0.23	0.41	0.54	0.68
	13	-0.73	-0.55	-0.41	-0.19	-0.06	0.02	0.10	0.23	0.41	0.54	0.68
	14	-0.66	-0.50	-0.37	-0.18	-0.05	0.03	0.10	0.22	0.41	0.54	0.68
	15	-0.59	-0.46	-0.35	-0.17	-0.04	0.03	0.10	0.23	0.42	0.54	0.68
16	3	-22.72	-13.55	-8.42	-3.73	-2.01	-1.27	-0.69	0.00	0.66	0.92	1.13
	4	-9.38	-5.89	-3.92	-1.89	-0.99	-0.58	-0.24	0.18	0.60	0.79	0.97
	5	-5.17	-3.45	-2.35	-1.15	-0.58	-0.32	-0.10	0.21	0.54	0.70	0.85
	6	-3.34	-2.34	-1.64	-0.79	-0.38	-0.19	-0.01	0.22	0.50	0.63	0.77
	7	-2.42	-1.68	-1.19	-0.57	-0.27	-0.11	0.03	0.23	0.46	0.59	0.73
	8	-1.81	-1.30	-0.93	-0.44	-0.19	-0.06	0.05	0.22	0.44	0.56	0.70
	9	-1.44	-1.05	-0.74	-0.36	-0.14	-0.03	0.07	0.22	0.42	0.54	0.68
	10	-1.18	-0.86	-0.61	-0.29	-0.11	-0.01	0.07	0.21	0.41	0.53	0.67
	11	-1.00	-0.72	-0.52	-0.25	-0.08	0.00	0.08	0.21	0.41	0.52	0.66
	12	-0.87	-0.64	-0.46	-0.21	-0.07	0.01	0.09	0.21	0.40	0.52	0.66
	13	-0.76	-0.56	-0.41	-0.19	-0.06	0.02	0.09	0.21	0.40	0.52	0.66
	14	-0.68	-0.51	-0.37	-0.17	-0.05	0.03	0.10	0.21	0.40	0.52	0.66
	15	-0.61	-0.46	-0.35	-0.16	-0.04	0.03	0.10	0.22	0.40	0.52	0.66
	16	-0.56	-0.44	-0.33	-0.15	-0.04	0.03	0.10	0.22	0.40	0.52	0.66

TABLE B-2 - (Continued)

Percentiles of the Distribution of $(\tilde{u}-u)/\tilde{b}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
17	3	-24.35	-13.91	-8.80	-3.79	-2.01	-1.27	-0.69	0.04	0.69	0.95	1.17
	4	-9.31	-6.05	-4.07	-1.92	-1.00	-0.60	-0.26	0.17	0.62	0.81	0.98
	5	-5.32	-3.60	-2.50	-1.21	-0.62	-0.34	-0.10	0.21	0.55	0.72	0.86
	6	-3.54	-2.43	-1.75	-0.85	-0.42	-0.21	-0.03	0.23	0.50	0.64	0.78
	7	-2.60	-1.82	-1.28	-0.62	-0.29	-0.13	0.02	0.22	0.46	0.59	0.71
	8	-1.94	-1.39	-0.98	-0.48	-0.21	-0.07	0.05	0.22	0.44	0.55	0.68
	9	-1.49	-1.11	-0.78	-0.38	-0.17	-0.05	0.06	0.21	0.42	0.53	0.66
	10	-1.25	-0.92	-0.66	-0.32	-0.13	-0.03	0.07	0.21	0.40	0.51	0.64
	11	-1.07	-0.77	-0.56	-0.27	-0.10	-0.01	0.07	0.21	0.39	0.50	0.63
	12	-0.90	-0.67	-0.50	-0.24	-0.08	-0.00	0.08	0.21	0.39	0.49	0.63
	13	-0.80	-0.59	-0.44	-0.21	-0.07	0.01	0.08	0.20	0.38	0.49	0.63
	14	-0.72	-0.54	-0.40	-0.19	-0.06	0.01	0.08	0.20	0.38	0.49	0.63
	15	-0.65	-0.49	-0.36	-0.18	-0.05	0.02	0.09	0.20	0.38	0.48	0.62
	16	-0.60	-0.46	-0.34	-0.16	-0.05	0.02	0.09	0.21	0.38	0.48	0.63
	17	-0.56	-0.43	-0.32	-0.16	-0.05	0.02	0.09	0.21	0.38	0.49	0.63
18	3	-25.92	-14.29	-8.73	-3.84	-2.01	-1.27	-0.69	0.02	0.69	0.97	1.21
	4	-9.67	-6.23	-4.12	-2.00	-1.04	-0.63	-0.29	0.17	0.62	0.83	1.00
	5	-5.55	-3.74	-2.59	-1.27	-0.66	-0.36	-0.12	0.21	0.56	0.72	0.87
	6	-3.67	-2.56	-1.77	-0.88	-0.43	-0.23	-0.03	0.22	0.51	0.65	0.78
	7	-2.64	-1.87	-1.31	-0.65	-0.31	-0.14	0.01	0.22	0.47	0.60	0.72
	8	-2.02	-1.42	-1.02	-0.51	-0.23	-0.09	0.04	0.22	0.44	0.55	0.68
	9	-1.62	-1.16	-0.83	-0.40	-0.17	-0.06	0.05	0.21	0.42	0.52	0.65
	10	-1.33	-0.95	-0.67	-0.33	-0.13	-0.03	0.06	0.21	0.40	0.50	0.63
	11	-1.13	-0.81	-0.58	-0.28	-0.11	-0.01	0.07	0.21	0.38	0.49	0.62
	12	-0.95	-0.69	-0.50	-0.24	-0.08	-0.00	0.08	0.20	0.37	0.48	0.61
	13	-0.84	-0.61	-0.45	-0.21	-0.07	0.01	0.08	0.20	0.37	0.48	0.61
	14	-0.74	-0.55	-0.40	-0.19	-0.06	0.01	0.08	0.20	0.37	0.48	0.60
	15	-0.67	-0.50	-0.37	-0.18	-0.05	0.02	0.09	0.20	0.37	0.48	0.61
	16	-0.61	-0.46	-0.35	-0.16	-0.05	0.02	0.09	0.20	0.37	0.48	0.60
	17	-0.57	-0.44	-0.32	-0.15	-0.04	0.02	0.09	0.20	0.37	0.48	0.61
	18	-0.54	-0.41	-0.31	-0.15	-0.04	0.02	0.09	0.20	0.37	0.48	0.61
19	3	-25.46	-14.84	-9.23	-4.11	-2.17	-1.37	-0.77	-0.02	0.68	0.96	1.22
	4	-10.39	-6.56	-4.35	-2.08	-1.13	-0.67	-0.31	0.15	0.62	0.83	1.01
	5	-5.84	-3.94	-2.68	-1.31	-0.69	-0.39	-0.14	0.21	0.57	0.72	0.87
	6	-3.76	-2.62	-1.84	-0.92	-0.46	-0.24	-0.05	0.22	0.51	0.65	0.78
	7	-2.77	-1.94	-1.38	-0.68	-0.33	-0.16	0.00	0.22	0.47	0.59	0.71
	8	-2.11	-1.50	-1.07	-0.53	-0.25	-0.10	0.04	0.22	0.43	0.55	0.67
	9	-1.67	-1.20	-0.87	-0.42	-0.19	-0.06	0.05	0.21	0.41	0.52	0.63
	10	-1.37	-0.98	-0.72	-0.35	-0.14	-0.04	0.06	0.21	0.40	0.50	0.62
	11	-1.13	-0.84	-0.61	-0.29	-0.11	-0.02	0.07	0.20	0.38	0.48	0.61
	12	-0.97	-0.72	-0.53	-0.25	-0.09	-0.01	0.07	0.20	0.37	0.48	0.60
	13	-0.84	-0.63	-0.46	-0.22	-0.08	0.00	0.08	0.20	0.36	0.47	0.59
	14	-0.75	-0.56	-0.41	-0.20	-0.07	0.01	0.08	0.20	0.36	0.47	0.59
	15	-0.68	-0.51	-0.37	-0.18	-0.06	0.01	0.08	0.19	0.36	0.47	0.59
	16	-0.62	-0.47	-0.35	-0.17	-0.05	0.02	0.08	0.19	0.36	0.46	0.59
	17	-0.58	-0.44	-0.33	-0.16	-0.05	0.02	0.08	0.19	0.36	0.46	0.58
	18	-0.54	-0.41	-0.31	-0.15	-0.04	0.02	0.08	0.19	0.36	0.46	0.59
	19	-0.51	-0.39	-0.30	-0.15	-0.04	0.02	0.09	0.20	0.36	0.46	0.59

TABLE B-2 - (Continued)

Percentiles of the Distribution of $(\tilde{u}-u)/\tilde{b}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
20	3	-26.67	-15.33	-9.32	-4.12	-2.18	-1.40	-0.79	-0.02	0.71	0.99	1.25
	4	-10.49	-6.64	-4.47	-2.15	-1.16	-0.70	-0.34	0.15	0.62	0.83	1.02
	5	-5.99	-4.00	-2.78	-1.37	-0.71	-0.41	-0.16	0.19	0.56	0.73	0.89
	6	-3.95	-2.73	-1.94	-0.96	-0.49	-0.26	-0.06	0.21	0.51	0.65	0.79
	7	-2.91	-2.04	-1.43	-0.72	-0.35	-0.17	-0.01	0.21	0.46	0.59	0.72
	8	-2.20	-1.55	-1.11	-0.55	-0.26	-0.11	0.02	0.21	0.43	0.55	0.68
	9	-1.72	-1.26	-0.91	-0.44	-0.19	-0.07	0.04	0.21	0.41	0.52	0.63
	10	-1.42	-1.03	-0.75	-0.37	-0.15	-0.05	0.05	0.20	0.39	0.49	0.60
	11	-1.19	-0.88	-0.63	-0.31	-0.13	-0.03	0.06	0.20	0.38	0.47	0.59
	12	-1.02	-0.76	-0.56	-0.26	-0.10	-0.01	0.07	0.20	0.36	0.46	0.59
	13	-0.89	-0.67	-0.49	-0.23	-0.08	-0.00	0.07	0.19	0.35	0.46	0.58
	14	-0.80	-0.59	-0.43	-0.20	-0.07	0.01	0.08	0.19	0.35	0.45	0.57
	15	-0.72	-0.53	-0.39	-0.19	-0.06	0.01	0.08	0.19	0.35	0.45	0.57
	16	-0.65	-0.49	-0.37	-0.17	-0.05	0.02	0.08	0.19	0.35	0.45	0.57
	17	-0.60	-0.46	-0.35	-0.16	-0.05	0.02	0.08	0.19	0.35	0.45	0.57
	18	-0.56	-0.43	-0.33	-0.15	-0.04	0.02	0.08	0.19	0.35	0.45	0.57
	19	-0.53	-0.41	-0.31	-0.14	-0.04	0.02	0.08	0.19	0.35	0.45	0.57
	20	-0.50	-0.40	-0.30	-0.14	-0.04	0.02	0.09	0.19	0.35	0.45	0.57
21	3	-27.07	-15.81	-9.63	-4.26	-2.27	-1.43	-0.79	-0.03	0.69	0.99	1.24
	4	-10.99	-6.70	-4.48	-2.22	-1.18	-0.72	-0.35	0.14	0.63	0.84	1.03
	5	-6.22	-4.17	-2.89	-1.40	-0.74	-0.43	-0.16	0.19	0.57	0.74	0.90
	6	-4.06	-2.83	-2.00	-0.99	-0.50	-0.26	-0.06	0.21	0.52	0.66	0.81
	7	-2.94	-2.07	-1.49	-0.73	-0.36	-0.17	-0.01	0.22	0.48	0.61	0.73
	8	-2.27	-1.62	-1.14	-0.57	-0.27	-0.12	0.02	0.22	0.44	0.56	0.67
	9	-1.81	-1.31	-0.95	-0.47	-0.21	-0.08	0.04	0.21	0.41	0.52	0.63
	10	-1.51	-1.10	-0.78	-0.39	-0.16	-0.05	0.05	0.20	0.39	0.49	0.60
	11	-1.27	-0.93	-0.66	-0.32	-0.14	-0.04	0.06	0.20	0.37	0.47	0.59
	12	-1.10	-0.80	-0.58	-0.28	-0.11	-0.02	0.06	0.19	0.36	0.46	0.57
	13	-0.94	-0.69	-0.50	-0.24	-0.09	-0.01	0.07	0.19	0.35	0.45	0.56
	14	-0.83	-0.61	-0.45	-0.21	-0.08	0.00	0.07	0.18	0.34	0.44	0.56
	15	-0.75	-0.56	-0.40	-0.19	-0.07	0.01	0.07	0.18	0.34	0.44	0.55
	16	-0.67	-0.51	-0.37	-0.18	-0.06	0.01	0.08	0.18	0.33	0.44	0.55
	17	-0.62	-0.47	-0.35	-0.17	-0.05	0.01	0.08	0.18	0.33	0.44	0.55
	18	-0.58	-0.44	-0.32	-0.15	-0.05	0.02	0.08	0.18	0.33	0.44	0.55
	19	-0.54	-0.41	-0.31	-0.15	-0.04	0.02	0.08	0.18	0.33	0.43	0.55
	20	-0.51	-0.39	-0.29	-0.14	-0.04	0.02	0.08	0.18	0.33	0.44	0.55
	21	-0.50	-0.38	-0.29	-0.14	-0.04	0.02	0.08	0.18	0.33	0.44	0.55

TABLE B-2 - (Continued)

Percentiles of the Distribution of $(\hat{u}-u)/\hat{b}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
22	3	-27.68	-16.16	-9.80	-4.35	-2.30	-1.48	-0.81	-0.02	0.72	1.03	1.29
	4	-11.07	-6.95	-4.70	-2.29	-1.22	-0.75	-0.37	0.14	0.65	0.86	1.06
	5	-6.39	-4.14	-2.89	-1.44	-0.76	-0.44	-0.16	0.19	0.58	0.75	0.92
	6	-4.16	-2.86	-2.02	-1.03	-0.52	-0.28	-0.07	0.21	0.53	0.68	0.82
	7	-3.12	-2.15	-1.53	-0.78	-0.39	-0.19	-0.02	0.22	0.49	0.61	0.[illegible]4
	8	-2.39	-1.68	-1.19	-0.61	-0.29	-0.13	0.01	0.21	0.45	0.56	0.68
	9	-1.91	-1.35	-0.98	-0.48	-0.22	-0.09	0.03	0.21	0.41	0.52	0.63
	10	-1.53	-1.11	-0.80	-0.39	-0.17	-0.06	0.05	0.20	0.39	0.49	0.60
	11	-1.31	-0.95	-0.69	-0.34	-0.14	-0.04	0.06	0.20	0.38	0.47	0.58
	12	-1.13	-0.81	-0.59	-0.29	-0.12	-0.02	0.06	0.19	0.36	0.46	0.57
	13	-0.96	-0.71	-0.52	-0.25	-0.10	-0.01	0.07	0.19	0.35	0.45	0.56
	14	-0.85	-0.64	-0.46	-0.23	-0.08	-0.01	0.07	0.18	0.35	0.44	0.55
	15	-0.76	-0.57	-0.42	-0.20	-0.07	0.00	0.07	0.18	0.34	0.44	0.54
	16	-0.70	-0.51	-0.38	-0.18	-0.06	0.01	0.07	0.18	0.34	0.43	0.54
	17	-0.64	-0.48	-0.35	-0.17	-0.05	0.01	0.07	0.18	0.33	0.43	0.54
	18	-0.58	-0.45	-0.33	-0.16	-0.05	0.01	0.08	0.18	0.33	0.43	0.54
	19	-0.55	-0.42	-0.31	-0.15	-0.05	0.01	0.07	0.18	0.33	0.43	0.54
	20	-0.52	-0.40	-0.29	-0.14	-0.04	0.02	0.08	0.18	0.33	0.43	0.54
	21	-0.49	-0.38	-0.28	-0.14	-0.04	0.02	0.08	0.18	0.33	0.43	0.54
	22	-0.47	-0.37	-0.27	-0.13	-0.04	0.02	0.08	0.18	0.33	0.43	0.54
23	3	-28.62	-16.34	-10.14	-4.63	-2.49	-1.97	-0.89	-0.07	0.72	1.02	1.30
	4	-12.02	-7.45	-5.06	-2.41	-1.31	-0.82	-0.40	0.13	0.65	0.88	1.08
	5	-6.81	-4.55	-3.12	-1.55	-0.84	-0.49	-0.20	0.18	0.58	0.77	0.93
	6	-4.44	-3.07	-2.18	-1.09	-0.56	-0.31	-0.09	0.21	0.53	0.69	0.82
	7	-3.19	-2.29	-1.60	-0.81	-0.40	-0.21	-0.03	0.21	0.49	0.62	0.75
	8	-2.53	-1.77	-1.26	-0.63	-0.31	-0.14	0.01	0.21	0.45	0.57	0.70
	9	-1.98	-1.42	-1.03	-0.51	-0.24	-0.10	0.02	0.21	0.42	0.54	0.65
	10	-1.61	-1.17	-0.85	-0.43	-0.19	-0.07	0.04	0.20	0.40	0.50	0.62
	11	-1.36	-1.00	-0.72	-0.36	-0.16	-0.05	0.05	0.19	0.38	0.48	0.59
	12	-1.17	-0.85	-0.61	-0.31	-0.13	-0.03	0.05	0.19	0.36	0.46	0.57
	13	-1.02	-0.75	-0.55	-0.27	-0.11	-0.02	0.06	0.19	0.35	0.45	0.55
	14	-0.89	-0.66	-0.48	-0.24	-0.09	-0.01	0.06	0.18	0.34	0.43	0.54
	15	-0.81	-0.60	-0.44	-0.21	-0.08	-0.00	0.07	0.18	0.34	0.43	0.54
	16	-0.73	-0.54	-0.40	-0.19	-0.07	0.00	0.07	0.18	0.33	0.42	0.53
	17	-0.66	-0.50	-0.36	-0.18	-0.06	0.01	0.07	0.17	0.33	0.42	0.53
	18	-0.61	-0.46	-0.34	-0.17	-0.05	0.01	0.07	0.17	0.32	0.42	0.53
	19	-0.57	-0.43	-0.32	-0.15	-0.05	0.01	0.07	0.17	0.32	0.42	0.53
	20	-0.53	-0.40	-0.30	-0.14	-0.05	0.01	0.07	0.17	0.32	0.41	0.53
	21	-0.49	-0.38	-0.29	-0.14	-0.04	0.01	0.07	0.17	0.32	0.41	0.53
	22	-0.47	-0.37	-0.28	-0.13	-0.04	0.02	0.07	0.17	0.32	0.41	0.53
	23	-0.46	-0.36	-0.27	-0.13	-0.04	0.02	0.07	0.17	0.32	0.41	0.53

TABLE B-2 - (Concluded)

Percentiles of the Distribution of $(\tilde{u}-u)/\tilde{b}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
24	3	-29.06	-16.48	-10.49	-4.64	-2.47	-1.57	-0.87	-0.04	0.70	1.01	1.26
	4	-11.67	-7.41	-4.95	-2.41	-1.30	-0.80	-0.39	0.13	0.65	0.87	1.08
	5	-6.87	-4.59	-3.13	-1.56	-0.83	-0.47	-0.19	0.18	0.58	0.76	0.92
	6	-4.57	-3.12	-2.19	-1.12	-0.57	-0.32	-0.10	0.20	0.52	0.68	0.82
	7	-3.28	-2.30	-1.65	-0.84	-0.42	-0.22	-0.04	0.20	0.48	0.62	0.75
	8	-2.50	-1.81	-1.29	-0.66	-0.32	-0.16	-0.01	0.20	0.44	0.56	0.68
	9	-1.96	-1.42	-1.04	-0.53	-0.25	-0.11	0.01	0.20	0.42	0.52	0.64
	10	-1.67	-1.20	-0.87	-0.43	-0.20	-0.08	0.03	0.19	0.39	0.49	0.60
	11	-1.40	-1.02	-0.73	-0.36	-0.16	-0.06	0.04	0.19	0.37	0.47	0.57
	12	-1.22	-0.88	-0.64	-0.31	-0.14	-0.04	0.05	0.18	0.36	0.45	0.55
	13	-1.06	-0.76	-0.56	-0.28	-0.12	-0.03	0.05	0.18	0.35	0.43	0.54
	14	-0.92	-0.68	-0.49	-0.24	-0.10	-0.02	0.06	0.18	0.33	0.42	0.53
	15	-0.82	-0.61	-0.44	-0.21	-0.08	-0.01	0.06	0.17	0.33	0.41	0.52
	16	-0.74	-0.56	-0.40	-0.19	-0.07	-0.00	0.06	0.17	0.32	0.41	0.51
	17	-0.69	-0.51	-0.37	-0.18	-0.07	0.00	0.06	0.17	0.31	0.40	0.51
	18	-0.63	-0.47	-0.35	-0.17	-0.06	0.00	0.07	0.17	0.31	0.40	0.51
	19	-0.59	-0.44	-0.33	-0.16	-0.05	0.01	0.07	0.17	0.31	0.40	0.51
	20	-0.55	-0.41	-0.31	-0.15	-0.05	0.01	0.07	0.16	0.31	0.40	0.51
	21	-0.51	-0.39	-0.30	-0.15	-0.04	0.02	0.07	0.17	0.31	0.40	0.50
	22	-0.49	-0.38	-0.29	-0.14	-0.04	0.02	0.07	0.17	0.31	0.39	0.51
	23	-0.47	-0.36	-0.28	-0.13	-0.04	0.02	0.07	0.17	0.31	0.40	0.51
	24	-0.45	-0.35	-0.27	-0.13	-0.04	0.02	0.07	0.17	0.31	0.40	0.51
25	3	-30.89	-17.48	-10.43	-4.67	-2.48	-1.60	-0.92	-0.08	0.72	1.05	1.33
	4	-11.74	-7.70	-5.09	-2.43	-1.32	-0.81	-0.41	0.12	0.67	0.90	1.11
	5	-6.69	-4.59	-3.20	-1.57	-0.83	-0.51	-0.21	0.18	0.59	0.78	0.96
	6	-4.60	-3.19	-2.21	-1.12	-0.61	-0.35	-0.12	0.20	0.54	0.69	0.84
	7	-3.44	-2.38	-1.68	-0.86	-0.45	-0.23	-0.05	0.20	0.48	0.63	0.76
	8	-2.58	-1.84	-1.33	-0.68	-0.34	-0.17	-0.01	0.20	0.45	0.57	0.70
	9	-2.08	-1.49	-1.09	-0.55	-0.27	-0.12	0.01	0.20	0.42	0.53	0.65
	10	-1.72	-1.25	-0.90	-0.45	-0.21	-0.08	0.03	0.20	0.40	0.50	0.61
	11	-1.44	-1.05	-0.76	-0.39	-0.17	-0.06	0.04	0.19	0.38	0.48	0.58
	12	-1.24	-0.90	-0.66	-0.33	-0.14	-0.04	0.05	0.19	0.36	0.45	0.56
	13	-1.07	-0.79	-0.57	-0.28	-0.12	-0.03	0.05	0.18	0.35	0.44	0.54
	14	-0.95	-0.70	-0.51	-0.25	-0.10	-0.02	0.06	0.18	0.34	0.42	0.52
	15	-0.84	-0.64	-0.46	-0.22	-0.09	-0.01	0.06	0.17	0.33	0.42	0.51
	16	-0.78	-0.58	-0.42	-0.20	-0.07	-0.00	0.06	0.17	0.32	0.41	0.51
	17	-0.71	-0.53	-0.38	-0.19	-0.07	0.00	0.07	0.17	0.32	0.41	0.51
	18	-0.64	-0.48	-0.36	-0.17	-0.06	0.01	0.07	0.17	0.32	0.40	0.51
	19	-0.60	-0.45	-0.33	-0.16	-0.05	0.01	0.07	0.17	0.31	0.40	0.50
	20	-0.54	-0.42	-0.31	-0.15	-0.05	0.01	0.07	0.17	0.31	0.40	0.50
	21	-0.52	-0.39	-0.29	-0.14	-0.04	0.01	0.07	0.17	0.31	0.40	0.51
	22	-0.48	-0.37	-0.28	-0.14	-0.04	0.01	0.07	0.17	0.31	0.40	0.50
	23	-0.47	-0.36	-0.27	-0.13	-0.04	0.02	0.07	0.17	0.31	0.39	0.50
	24	-0.45	-0.35	-0.26	-0.13	-0.04	0.02	0.07	0.17	0.31	0.39	0.50
	25	-0.43	-0.34	-0.26	-0.13	-0.03	0.02	0.07	0.17	0.31	0.40	0.50

TABLE B-3

Percentiles of the Distribution of $V_{.90}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
3	3	0.75	1.10	1.43	2.18	2.88	3.40	4.06	5.50	8.99	13.16	20.93
4	3	0.78	1.16	1.49	2.18	2.82	3.33	3.96	5.38	9.03	13.07	20.23
	4	0.87	1.16	1.46	2.06	2.60	2.99	3.45	4.40	6.47	8.39	11.66
5	3	0.78	1.18	1.51	2.17	2.79	3.27	3.87	5.24	8.78	12.58	20.38
	4	0.97	1.23	1.51	2.09	2.61	2.99	3.44	4.40	6.49	8.48	11.73
	5	0.97	1.23	1.49	2.02	2.49	2.82	3.20	3.93	5.48	6.73	8.66
6	3	0.73	1.18	1.53	2.15	2.73	3.18	3.74	4.98	8.24	11.74	18.65
	4	1.00	1.28	1.55	2.10	2.60	2.98	3.41	4.30	6.33	8.18	11.39
	5	1.02	1.29	1.54	2.05	2.50	2.82	3.21	3.94	5.42	6.73	8.89
	6	1.02	1.27	1.53	2.01	2.42	2.70	3.04	3.67	4.86	5.83	7.31
7	3	0.64	1.18	1.53	2.13	2.66	3.08	3.60	4.79	7.80	11.12	17.54
	4	1.04	1.31	1.58	2.10	2.57	2.91	3.23	4.21	6.16	7.89	10.90
	5	1.08	1.33	1.57	2.06	2.49	2.80	3.15	3.87	5.36	6.68	8.44
	6	1.08	1.32	1.56	2.03	2.42	2.70	3.01	3.63	4.86	5.82	7.23
	7	1.08	1.32	1.55	2.00	2.37	2.62	2.90	3.44	4.46	5.25	6.37
8	3	0.49	1.13	1.52	2.11	2.62	3.01	3.48	4.62	7.51	10.67	16.36
	4	1.04	1.33	1.60	2.10	2.56	2.88	3.27	4.10	5.96	7.79	10.75
	5	1.11	1.36	1.60	2.08	2.49	2.78	3.12	3.82	5.28	6.50	8.62
	6	1.13	1.36	1.59	2.05	2.43	2.71	3.02	3.62	4.83	5.83	7.18
	7	1.12	1.36	1.58	2.03	2.38	2.64	2.93	3.46	4.49	5.31	6.40
	8	1.12	1.36	1.58	2.01	2.34	2.57	2.83	3.32	4.21	4.90	5.84
9	3	0.42	1.12	1.51	2.09	2.57	2.95	3.40	4.43	7.14	10.21	15.61
	4	1.06	1.36	1.61	2.10	2.52	2.84	3.21	4.00	5.77	7.39	10.26
	5	1.17	1.41	1.63	2.08	2.47	2.76	3.08	3.76	5.13	6.34	8.13
	6	1.19	1.41	1.62	2.06	2.43	2.70	2.99	3.59	4.74	5.67	7.06
	7	1.19	1.41	1.62	2.04	2.39	2.64	2.91	3.45	4.48	5.28	6.46
	8	1.19	1.40	1.61	2.02	2.36	2.59	2.84	3.34	4.26	4.95	5.94
	9	1.19	1.40	1.60	2.00	2.33	2.55	2.78	3.27	4.04	4.66	5.50

TABLE B-3 - (Continued)

Percentiles of the Distribution of $V_{.90}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
10	3	0.09	0.99	1.46	2.05	2.51	2.84	3.27	4.25	6.75	9.36	14.88
	4	0.99	1.34	1.62	2.08	2.48	2.77	3.13	3.90	5.56	7.17	9.60
	5	1.17	1.42	1.64	2.07	2.45	2.71	3.02	3.67	[illegible]	6.13	8.02
	6	1.20	1.43	1.64	2.05	2.41	2.66	2.94	3.53	4.67	5.59	6.99
	7	[illegible]	1.43	1.64	2.04	2.38	2.62	2.88	3.41	4.41	5.18	6.29
	8	1.21	1.43	1.63	2.02	2.35	2.58	2.83	3.31	4.27	4.91	5.83
	9	1.21	1.42	1.63	2.01	2.32	2.54	2.77	3.22	4.03	4.63	5.51
	10	1.21	1.42	1.62	1.99	2.30	2.50	2.72	3.13	3.86	4.41	5.16
11	3	-0.09	0.90	1.42	2.01	2.45	2.77	3.17	4.07	6.41	9.11	14.47
	4	0.97	1.35	1.61	2.06	2.44	2.73	3.06	3.79	5.46	7.04	9.98
	5	1.18	1.43	1.64	2.05	2.41	2.68	2.98	3.60	4.90	6.07	7.83
	6	1.24	1.45	1.64	2.04	2.38	2.63	2.91	3.46	4.58	5.52	6.96
	7	1.25	1.45	1.64	2.03	[illegible]	2.59	2.86	3.36	4.36	5.16	6.34
	8	1.25	1.45	1.64	2.01	2.33	2.56	2.80	3.28	4.15	4.87	5.82
	9	1.25	1.44	1.64	2.00	2.31	2.53	2.76	3.21	4.01	4.63	5.54
	10	1.25	1.44	1.64	1.99	2.29	2.49	2.71	3.14	3.87	4.44	5.23
	11	1.25	1.45	1.63	1.98	2.28	2.46	2.67	3.06	3.76	4.26	4.94
12	3	-0.38	0.75	1.37	1.98	2.41	2.71	3.08	3.89	6.00	8.40	12.96
	4	0.95	1.34	1.60	2.05	2.42	2.69	3.00	3.67	5.17	6.60	9.07
	5	1.20	1.44	1.66	2.05	2.40	2.65	2.93	3.52	4.72	5.79	7.35
	6	1.26	1.46	1.67	2.04	2.38	2.62	2.88	3.39	4.41	5.31	6.61
	7	1.28	1.47	1.67	2.03	2.36	2.58	2.82	3.30	4.21	4.98	6.09
	8	1.28	1.47	1.66	2.02	2.34	2.54	2.78	3.22	4.06	4.75	5.71
	9	1.27	1.46	1.66	2.01	2.31	2.52	2.74	3.16	3.94	4.53	5.40
	10	1.27	1.47	1.65	2.00	2.30	2.49	2.70	3.11	3.82	4.37	5.11
	11	1.27	1.46	1.64	2.00	2.28	2.47	2.67	3.05	3.72	4.23	4.88
	12	1.28	1.47	1.64	1.99	2.27	2.44	2.63	3.00	3.62	4.07	4.68
13	3	-0.45	0.72	1.34	1.99	2.40	2.69	3.04	3.85	5.88	8.16	12.45
	4	0.88	1.31	1.60	2.06	2.42	2.68	2.98	3.64	5.10	6.45	8.82
	5	1.20	1.45	1.67	2.07	2.40	2.64	2.92	3.49	4.71	5.75	7.32
	6	1.27	1.48	1.68	2.07	2.38	2.61	2.86	3.38	4.63	5.30	6.49
	7	1.30	1.49	1.68	2.06	2.36	2.58	2.82	3.30	4.23	4.96	6.02
	8	1.30	1.49	1.68	2.04	2.34	2.55	2.78	3.22	4.06	4.73	5.63
	9	1.30	1.49	1.68	2.03	2.33	2.52	2.75	3.16	3.94	4.55	5.32
	10	1.30	1.49	1.68	2.03	2.31	2.50	2.72	3.12	3.83	4.37	5.11
	11	1.30	1.49	1.68	2.02	2.30	2.48	2.69	3.06	3.74	4.23	4.90
	12	1.30	1.49	1.67	2.01	2.28	2.46	2.65	3.02	3.65	4.09	4.73
	13	1.30	1.49	1.67	2.01	2.27	2.44	2.62	2.97	3.57	3.97	4.51

TABLE B-3 - (Continued)

Percentiles of the Distribution of $V_{.90}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
14	3	-0.81	0.57	1.25	1.93	2.34	2.62	2.95	3.70	5.56	7.69	11.56
	4	0.83	1.29	1.59	2.03	2.37	2.62	2.92	3.54	4.93	6.17	8.28
	5	1.18	1.46	1.67	2.05	2.36	2.60	2.86	3.42	4.58	5.54	6.95
	6	1.28	1.51	1.69	2.05	2.35	2.57	2.82	3.34	4.33	5.12	6.27
	7	1.32	1.52	1.69	2.04	2.34	2.55	2.78	3.27	4.15	4.82	5.75
	8	1.33	1.52	1.69	2.03	2.33	2.52	2.74	3.19	4.03	4.61	5.47
	9	1.33	1.52	1.69	2.03	2.31	2.50	2.71	3.14	3.90	4.45	5.18
	10	1.33	1.51	1.68	2.02	2.30	2.48	2.70	3.09	3.78	4.30	4.94
	11	1.33	1.51	1.68	2.01	2.28	2.47	2.67	3.05	3.71	4.20	4.79
	12	1.33	1.51	1.68	2.01	2.27	2.45	2.65	3.01	3.64	4.09	4.67
	13	1.33	1.51	1.68	2.00	2.26	2.43	2.62	2.97	3.55	3.98	4.51
	14	1.33	1.51	1.68	2.00	2.25	2.42	2.60	2.93	3.46	3.85	4.36
15	3	-1.05	0.43	1.19	1.91	2.33	2.60	2.91	3.64	5.39	7.23	10.78
	4	0.77	1.26	1.59	2.03	2.37	2.61	2.89	3.49	4.78	5.95	7.94
	5	1.15	1.44	1.67	2.06	2.37	2.59	2.85	3.38	4.43	5.36	6.85
	6	1.29	1.50	1.69	2.06	2.36	2.57	2.81	3.30	4.22	4.97	6.19
	7	1.33	1.52	1.70	2.06	2.35	2.55	2.78	3.23	4.08	4.72	5.77
	8	1.34	1.52	1.70	2.05	2.33	2.53	2.74	3.17	3.95	4.57	5.40
	9	1.35	1.52	1.69	2.04	2.32	2.51	2.72	3.13	3.85	4.40	5.16
	10	1.35	1.52	1.69	2.04	2.31	2.49	2.69	3.09	3.76	4.26	4.95
	11	1.35	1.52	1.69	2.03	2.30	2.48	2.67	3.04	3.69	4.15	4.76
	12	1.34	1.52	1.69	2.02	2.28	2.46	2.64	3.00	3.67	4.08	4.62
	13	1.35	1.52	1.68	2.01	2.27	2.44	2.63	2.96	3.55	3.98	4.51
	14	1.35	1.51	1.69	2.01	2.27	2.43	2.60	2.93	3.49	3.89	4.39
	15	1.35	1.52	1.68	2.01	2.25	2.41	2.58	2.89	3.41	3.77	4.23
16	3	-1.38	0.25	1.10	1.90	2.31	2.57	2.88	3.57	5.22	7.07	10.49
	4	0.74	1.23	1.58	2.03	2.37	2.59	2.87	3.45	4.72	5.90	7.94
	5	1.17	1.45	1.68	2.06	2.37	2.59	2.84	3.36	4.42	5.33	6.73
	6	1.30	1.52	1.71	2.07	2.36	2.57	2.81	3.28	4.20	4.98	6.18
	7	1.35	1.53	1.72	2.06	2.35	2.56	2.77	3.21	4.05	4.74	5.81
	8	1.37	1.54	1.72	2.06	2.34	2.53	2.75	3.16	3.94	4.56	5.48
	9	1.37	1.54	1.72	2.05	2.33	2.51	2.72	3.11	3.84	4.38	5.17
	10	1.37	1.54	1.71	2.04	2.31	2.50	2.70	3.08	3.74	4.24	4.97
	11	1.37	1.54	1.71	2.04	2.30	2.48	2.68	3.04	3.67	4.13	4.79
	12	1.37	1.54	1.71	2.03	2.29	2.46	2.65	3.00	3.60	4.05	4.65
	13	1.37	1.54	1.71	2.03	2.28	2.45	2.63	2.97	3.52	3.94	4.49
	14	1.37	1.54	1.71	2.02	2.27	2.43	2.61	2.94	3.48	3.87	4.38
	15	1.37	1.54	1.71	2.02	2.26	2.42	2.59	2.90	3.41	3.79	4.26
	16	1.38	1.54	1.71	2.02	2.26	2.40	2.56	2.87	3.36	3.71	4.16

TABLE B-3 - (Continued)

Percentiles of the Distribution of $V_{.90}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
17	3	-1.56	0.10	1.03	1.86	2.26	2.50	2.79	3.41	4.96	6.54	9.76
	4	0.61	1.18	1.56	2.01	2.32	2.54	2.80	3.33	4.48	5.56	7.33
	5	1.08	1.43	1.67	2.04	2.33	2.54	2.78	3.27	4.24	5.09	6.35
	6	1.28	1.52	1.71	2.05	2.33	2.52	2.75	3.20	4.07	4.76	5.81
	7	1.34	1.54	1.72	2.05	2.32	2.51	2.72	3.14	3.95	4.56	5.44
	8	1.36	1.55	1.72	2.04	2.31	2.49	2.69	3.10	3.85	4.41	5.19
	9	1.37	1.56	1.72	2.04	2.30	2.48	2.67	3.05	3.76	4.28	4.97
	10	1.38	1.55	1.72	2.03	2.29	2.47	2.65	3.01	3.69	4.16	4.80
	11	1.38	1.55	1.72	2.03	2.28	2.45	2.64	2.98	3.62	4.05	4.66
	12	1.38	1.55	1.71	2.02	2.27	2.44	2.62	2.95	3.56	3.98	4.53
	13	1.38	1.55	1.71	2.02	2.26	2.42	2.60	2.92	3.50	3.90	4.45
	14	1.38	1.55	1.71	2.01	2.25	2.42	2.58	2.90	3.45	3.83	4.31
	15	1.38	1.55	1.71	2.01	2.25	2.40	2.57	2.88	3.40	3.76	4.19
	16	1.38	1.55	1.71	2.00	2.24	2.39	2.55	2.85	3.35	3.70	4.10
	17	1.38	1.55	1.71	2.00	2.24	2.38	2.53	2.82	3.29	3.62	4.04
18	3	-1.61	0.11	1.01	1.85	2.26	2.50	2.77	3.38	4.81	6.43	9.64
	4	0.66	1.15	1.56	2.01	2.32	2.54	2.79	3.32	4.44	5.52	7.38
	5	1.10	1.45	1.69	2.05	2.34	2.54	2.77	3.26	4.22	5.04	6.40
	6	1.30	1.53	1.73	2.06	2.34	2.53	2.75	3.19	4.06	4.73	5.79
	7	1.37	1.56	1.74	2.06	2.33	2.52	2.73	3.14	3.94	4.55	5.41
	8	1.40	1.57	1.74	2.06	2.32	2.50	2.71	3.09	3.84	4.39	5.15
	9	1.41	1.58	1.74	2.05	2.31	2.49	2.68	3.06	3.75	4.25	4.99
	10	1.41	1.58	1.74	2.05	2.30	2.48	2.67	3.03	3.67	4.14	4.80
	11	1.41	1.58	1.74	2.05	2.29	2.46	2.65	3.00	3.63	4.05	4.63
	12	1.41	1.58	1.74	2.04	2.28	2.45	2.63	2.97	3.56	3.97	4.51
	13	1.41	1.57	1.74	2.04	2.28	2.44	2.61	2.94	3.50	3.91	4.40
	14	1.41	1.57	1.73	2.03	2.27	2.43	2.60	2.92	3.45	3.84	4.30
	15	1.41	1.57	1.73	2.03	2.26	2.42	2.59	2.89	3.40	3.75	4.21
	16	1.41	1.57	1.73	2.02	2.26	2.41	2.57	2.87	3.36	3.71	4.14
	17	1.41	1.58	1.74	2.02	2.25	2.40	2.55	2.84	3.31	3.66	4.04
	18	1.42	1.58	1.74	2.02	2.24	2.39	2.54	2.82	3.27	3.59	3.97
19	3	-2.13	-0.16	0.83	1.81	2.23	2.47	2.72	3.29	4.68	6.09	9.01
	4	0.43	1.11	1.51	1.99	2.30	2.51	2.75	3.25	4.32	5.33	7.04
	5	1.09	1.44	1.67	2.04	2.33	2.52	2.74	3.20	4.13	4.94	6.21
	6	1.30	1.53	1.72	2.05	2.33	2.52	2.72	3.15	3.98	4.67	5.67
	7	1.38	1.56	1.74	2.05	2.32	2.50	2.70	3.11	3.87	4.45	5.33
	8	1.41	1.58	1.74	2.05	2.31	2.49	2.68	3.07	3.79	4.30	5.09
	9	1.42	1.58	1.74	2.05	2.31	2.48	2.67	3.03	3.71	4.23	4.88
	10	1.43	1.58	1.74	2.05	2.30	2.46	2.65	3.00	3.64	4.11	4.73
	11	1.43	1.58	1.74	2.04	2.29	2.45	2.63	2.98	3.57	4.02	4.63
	12	1.43	1.58	1.74	2.04	2.28	2.45	2.62	2.95	3.52	3.94	4.51
	13	1.43	1.58	1.74	2.03	2.27	2.43	2.60	2.93	3.48	3.88	4.41
	14	1.43	1.58	1.73	2.03	2.26	2.42	2.59	2.90	3.43	3.81	4.29
	15	1.42	1.58	1.74	2.03	2.26	2.41	2.57	2.88	3.39	3.75	4.22
	16	1.42	1.58	1.73	2.03	2.25	2.40	2.56	2.86	3.36	3.70	4.10
	17	1.42	1.58	1.74	2.02	2.25	2.40	2.55	2.84	3.32	3.65	4.06
	18	1.42	1.58	1.74	2.02	2.24	2.38	2.54	2.81	3.29	3.59	3.99
	19	1.43	1.59	1.74	2.02	2.24	2.38	2.52	2.78	3.24	3.53	3.90

TABLE B-3 - (Continued)

Percentiles of the Distribution of $V_{.90}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
20	3	-2.27	-0.19	0.82	1.79	2.20	2.44	2.69	3.23	4.53	6.04	9.01
	4	0.46	1.10	1.51	1.99	2.29	2.49	2.72	3.21	4.24	5.24	7.00
	5	1.07	1.42	1.67	2.04	2.31	2.50	2.72	3.16	4.04	4.82	6.16
	6	1.30	1.54	1.73	2.06	2.32	2.50	2.71	3.13	3.90	4.55	5.68
	7	1.38	1.58	1.74	2.06	2.31	2.49	2.69	3.08	3.80	4.41	5.33
	8	1.42	1.59	1.75	2.06	2.30	2.48	2.67	3.04	3.71	4.29	5.06
	9	1.43	1.60	1.75	2.06	2.29	2.46	2.65	3.01	3.65	4.15	4.85
	10	1.44	1.60	1.75	2.05	2.29	2.45	2.64	2.98	3.59	4.05	4.71
	11	1.44	1.60	1.75	2.05	2.28	2.44	2.63	2.95	3.54	3.97	4.57
	12	1.44	1.60	1.75	2.04	2.28	2.43	2.61	2.92	3.49	3.92	4.46
	13	1.44	1.60	1.75	2.04	2.27	2.42	2.59	2.90	3.44	3.84	4.39
	14	1.44	1.60	1.75	2.04	2.26	2.41	2.58	2.88	3.40	3.77	4.29
	15	1.43	1.60	1.75	2.03	2.26	2.40	2.56	2.86	3.37	3.72	4.20
	16	1.44	1.60	1.75	2.03	2.25	2.40	2.55	2.84	3.33	3.67	4.12
	17	1.44	1.60	1.75	2.03	2.24	2.39	2.54	2.82	3.29	3.61	4.04
	18	1.44	1.60	1.74	2.02	2.24	2.38	2.53	2.81	3.25	3.57	3.97
	19	1.44	1.60	1.75	2.02	2.24	2.37	2.52	2.78	3.21	3.52	3.92
	20	1.44	1.60	1.75	2.02	2.23	2.36	2.51	2.76	3.17	3.47	3.84
21	3	-2.70	-0.41	0.72	1.74	2.17	2.41	2.66	3.17	4.40	5.76	8.25
	4	0.32	1.04	1.47	1.97	2.28	2.47	2.70	3.16	4.16	5.06	6.51
	5	1.05	1.41	1.66	2.03	2.30	2.49	2.70	3.14	4.01	4.75	5.88
	6	1.29	1.53	1.72	2.05	2.31	2.49	2.69	3.10	3.87	4.53	5.47
	7	1.39	1.58	1.75	2.05	2.31	2.48	2.68	3.06	3.78	4.35	5.13
	8	1.43	1.60	1.76	2.06	2.30	2.47	2.66	3.03	3.71	4.23	4.95
	9	1.45	1.60	1.76	2.05	2.30	2.46	2.64	3.00	3.65	4.13	4.78
	10	1.45	1.60	1.76	2.05	2.29	2.45	2.63	2.97	3.59	4.05	4.64
	11	1.45	1.60	1.76	2.05	2.28	2.44	2.62	2.95	3.53	3.97	4.54
	12	1.45	1.60	1.76	2.04	2.28	2.43	2.61	2.93	3.49	3.90	4.44
	13	1.45	1.60	1.76	2.04	2.27	2.42	2.60	2.90	3.45	3.83	4.33
	14	1.45	1.60	1.75	2.03	2.26	2.41	2.59	2.89	3.41	3.79	4.29
	15	1.45	1.60	1.75	2.03	2.26	2.40	2.57	2.86	3.37	3.72	4.19
	16	1.46	1.60	1.75	2.03	2.25	2.40	2.56	2.85	3.34	3.66	4.11
	17	1.46	1.60	1.75	2.03	2.25	2.39	2.54	2.83	3.30	3.61	4.04
	18	1.46	1.61	1.75	2.02	2.24	2.38	2.54	2.81	3.27	3.56	3.97
	19	1.46	1.61	1.75	2.02	2.24	2.38	2.52	2.79	3.23	3.53	3.92
	20	1.46	1.61	1.75	2.02	2.23	2.37	2.51	2.77	3.19	3.48	3.85
	21	1.47	1.62	1.76	2.02	2.23	2.36	2.50	2.75	3.15	3.43	3.77

TABLE B-3 - (Continued)

Percentiles of the Distribution of $V_{.90}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
22	3	-3.15	-0.61	0.61	1.71	2.16	2.39	2.63	3.13	4.31	5.49	7.84
	4	0.26	0.98	1.45	1.96	2.25	2.45	2.67	3.13	4.10	4.96	6.51
	5	1.03	1.42	1.66	2.02	2.29	2.47	2.68	3.10	3.95	4.65	5.70
	6	1.27	1.54	1.73	2.04	2.30	2.47	2.67	3.07	3.84	4.43	5.32
	7	1.38	1.59	1.76	2.05	2.30	2.47	2.66	3.05	3.76	4.32	5.09
	8	1.43	1.61	1.76	2.05	2.30	2.46	2.65	3.01	3.68	4.17	4.87
	9	1.46	1.62	1.77	2.05	2.29	2.45	2.64	2.98	3.61	4.07	4.69
	10	1.47	1.62	1.77	2.05	2.29	2.44	2.62	2.95	3.57	3.99	4.55
	11	1.48	1.62	1.77	2.05	2.28	2.43	2.61	2.93	3.51	3.92	4.44
	12	1.48	1.62	1.77	2.04	2.27	2.42	2.60	2.92	3.47	3.86	4.37
	13	1.48	1.62	1.77	2.04	2.27	2.41	2.58	2.89	3.43	3.80	4.27
	14	1.48	1.62	1.77	2.04	2.26	2.41	2.57	2.87	3.38	3.75	4.22
	15	1.47	1.62	1.76	2.04	2.26	2.40	2.56	2.85	3.34	3.69	4.15
	16	1.47	1.62	1.76	2.03	2.25	2.40	2.55	2.84	3.31	3.63	4.07
	17	1.46	1.62	1.77	2.03	2.25	2.39	2.54	2.82	3.28	3.61	4.01
	18	1.47	1.62	1.76	2.03	2.24	2.38	2.53	2.80	3.25	3.57	3.95
	19	1.47	1.62	1.76	2.03	2.24	2.38	2.52	2.78	3.22	3.53	3.90
	20	1.47	1.62	1.76	2.03	2.23	2.37	2.51	2.77	3.20	3.47	3.85
	21	1.48	1.62	1.77	2.03	2.23	2.36	2.50	2.76	3.17	3.44	3.78
	22	1.48	1.63	1.77	2.03	2.23	2.36	2.49	2.74	3.13	3.39	3.71
23	3	-3.04	-0.75	0.49	1.67	2.13	2.37	2.60	3.07	4.19	5.40	7.86
	4	0.14	0.94	1.41	1.94	2.25	2.44	2.65	3.08	4.01	4.90	6.52
	5	0.95	1.37	1.64	2.01	2.29	2.46	2.66	3.06	3.89	4.60	5.82
	6	1.24	1.51	1.72	2.05	2.30	2.47	2.65	3.04	3.77	4.40	5.31
	7	1.37	1.58	1.75	2.06	2.30	2.47	2.64	3.01	3.68	4.26	5.07
	8	1.44	1.60	1.76	2.06	2.29	2.46	2.63	2.98	3.62	4.12	4.86
	9	1.46	1.62	1.77	2.06	2.29	2.45	2.62	2.95	3.57	4.04	4.69
	10	1.47	1.62	1.77	2.06	2.28	2.44	2.61	2.93	3.51	3.95	4.55
	11	1.48	1.63	1.77	2.05	2.28	2.43	2.60	2.91	3.46	3.88	4.42
	12	1.48	1.62	1.77	2.05	2.27	2.42	2.59	2.89	3.42	3.80	4.33
	13	1.48	1.62	1.77	2.05	2.27	2.41	2.57	2.87	3.39	3.74	4.22
	14	1.48	1.62	1.76	2.04	2.26	2.41	2.56	2.86	3.36	3.71	4.16
	15	1.48	1.62	1.76	2.04	2.26	2.40	2.55	2.84	3.33	3.67	4.08
	16	1.48	1.62	1.76	2.04	2.25	2.39	2.54	2.82	3.30	3.62	4.04
	17	1.48	1.62	1.76	2.03	2.25	2.39	2.53	2.81	3.27	3.59	3.99
	18	1.48	1.62	1.76	2.03	2.25	2.38	2.52	2.79	3.25	3.55	3.94
	19	1.47	1.62	1.76	2.03	2.24	2.37	2.52	2.78	3.22	3.52	3.89
	20	1.48	1.62	1.76	2.03	2.24	2.37	2.51	2.77	3.18	3.48	3.84
	21	1.48	1.62	1.76	2.03	2.23	2.36	2.50	2.75	3.16	3.44	3.79
	22	1.48	1.62	1.77	2.03	2.23	2.36	2.49	2.74	3.13	3.41	3.73
	23	1.49	1.63	1.77	2.03	2.22	2.35	2.48	2.72	3.11	3.37	3.68

TABLE B-3 - (Concluded)

Percentiles of the Distribution of $V_{.90}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
24	3	-3.60	-1.00	0.45	1.62	2.11	2.36	2.59	3.05	4.11	5.20	7.27
	4	0.11	0.91	1.38	1.94	2.24	2.43	2.64	3.06	3.96	4.75	6.16
	5	0.91	1.36	1.64	2.02	2.28	2.45	2.66	3.05	3.85	4.50	5.60
	6	1.24	1.52	1.72	2.05	2.29	2.46	2.66	3.03	3.75	4.34	5.22
	7	1.38	1.58	1.75	2.06	2.29	2.45	2.65	3.00	3.68	4.22	4.95
	8	1.43	1.61	1.77	2.07	2.29	2.45	2.63	2.98	3.60	4.10	4.77
	9	1.46	1.62	1.77	2.07	2.29	2.44	2.62	2.95	3.56	4.01	4.63
	10	1.47	1.63	1.78	2.06	2.28	2.44	2.61	2.93	3.51	3.94	4.51
	11	1.48	1.63	1.78	2.06	2.28	2.43	2.59	2.91	3.46	3.87	4.38
	12	1.48	1.63	1.77	2.06	2.27	2.42	2.58	2.89	3.43	3.81	4.29
	13	1.48	1.63	1.77	2.05	2.27	2.42	2.57	2.87	3.38	3.76	4.23
	14	1.48	1.63	1.77	2.05	2.27	2.41	2.57	2.85	3.34	3.70	4.14
	15	1.48	1.63	1.77	2.05	2.26	2.40	2.56	2.84	3.31	3.65	4.07
	16	1.48	1.63	1.77	2.05	2.26	2.40	2.55	2.82	3.28	3.61	4.02
	17	1.48	1.63	1.77	2.05	2.25	2.39	2.54	2.81	3.26	3.57	3.96
	18	1.48	1.63	1.77	2.04	2.25	2.39	2.53	2.79	3.23	3.54	3.92
	19	1.48	1.63	1.77	2.04	2.25	2.38	2.52	2.78	3.20	3.50	3.89
	20	1.48	1.63	1.77	2.04	2.25	2.38	2.51	2.77	3.18	3.47	3.82
	21	1.48	1.63	1.77	2.04	2.24	2.37	2.51	2.75	3.14	3.42	3.78
	22	1.48	1.63	1.78	2.04	2.23	2.36	2.50	2.74	3.12	3.39	3.75
	23	1.49	1.63	1.78	2.04	2.23	2.36	2.49	2.73	3.10	3.36	3.69
	24	1.49	1.64	1.78	2.04	2.23	2.35	2.48	2.71	3.07	3.32	3.65
25	3	-3.88	-1.13	0.28	1.60	2.10	2.34	2.57	3.00	3.98	4.98	7.02
	4	0.04	0.86	1.35	1.94	2.24	2.43	2.62	3.02	3.87	4.64	6.06
	5	0.92	1.36	1.63	2.02	2.28	2.45	2.64	3.01	3.76	4.41	5.44
	6	1.24	1.51	1.72	2.05	2.30	2.46	2.64	3.00	3.69	4.26	5.13
	7	1.38	1.59	1.76	2.06	2.31	2.46	2.63	2.98	3.63	4.14	4.87
	8	1.44	1.62	1.78	2.06	2.30	2.45	2.63	2.96	3.56	4.04	4.72
	9	1.47	1.64	1.79	2.07	2.30	2.45	2.61	2.93	3.52	3.96	4.61
	10	1.49	1.65	1.79	2.06	2.30	2.44	2.60	2.92	3.47	3.89	4.47
	11	1.50	1.65	1.79	2.06	2.29	2.43	2.59	2.90	3.43	3.82	4.36
	12	1.50	1.65	1.79	2.06	2.28	2.43	2.58	2.88	3.40	3.76	4.31
	13	1.50	1.65	1.79	2.06	2.28	2.42	2.57	2.86	3.36	3.72	4.22
	14	1.50	1.65	1.79	2.05	2.27	2.41	2.57	2.85	3.33	3.69	4.15
	15	1.50	1.65	1.79	2.05	2.26	2.40	2.56	2.84	3.30	3.64	4.06
	16	1.50	1.65	1.79	2.05	2.26	2.40	2.55	2.82	3.27	3.59	4.01
	17	1.50	1.65	1.79	2.05	2.25	2.39	2.54	2.81	3.26	3.57	3.95
	18	1.50	1.65	1.79	2.05	2.25	2.39	2.53	2.79	3.22	3.52	3.91
	19	1.50	1.65	1.78	2.05	2.25	2.38	2.52	2.78	3.21	3.49	3.84
	20	1.50	1.65	1.79	2.04	2.25	2.37	2.51	2.76	3.18	3.46	3.79
	21	1.50	1.65	1.79	2.04	2.24	2.37	2.50	2.75	3.16	3.43	3.76
	22	1.51	1.65	1.79	2.04	2.24	2.36	2.50	2.74	3.14	3.40	3.72
	23	1.51	1.65	1.79	2.04	2.23	2.36	2.49	2.72	3.12	3.38	3.70
	24	1.51	1.66	1.79	2.04	2.23	2.35	2.48	2.71	3.09	3.35	3.66
	25	1.52	1.66	1.80	2.03	2.22	2.35	2.47	2.70	3.07	3.31	3.61

TABLE B-4

Percentiles of the Distribution of $V_{.95}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
3	3	1.26	1.64	2.04	2.94	3.83	4.49	5.33	7.20	11.75	17.21	27.32
4	3	1.38	1.73	2.11	2.98	3.82	4.47	5.30	7.19	12.17	17.55	27.59
	4	1.36	1.69	2.04	2.78	3.44	3.95	4.51	5.73	8.40	10.88	15.06
5	3	1.44	1.79	2.16	3.00	3.81	4.46	5.27	7.16	12.07	17.36	28.30
	4	1.45	1.76	2.10	2.82	3.49	3.98	4.56	5.80	8.56	11.14	15.51
	5	1.44	1.74	2.06	2.72	3.29	3.70	4.17	5.11	7.06	8.68	11.14
6	3	1.48	1.83	2.20	2.99	3.77	4.37	5.15	6.93	11.53	16.66	26.85
	4	1.52	1.83	2.15	2.84	3.49	3.98	4.55	5.75	8.47	10.95	15.32
	5	1.51	1.81	2.12	2.76	3.33	3.73	4.21	5.17	7.08	8.82	11.58
	6	1.50	1.80	2.10	2.70	3.20	3.56	3.97	4.77	6.27	7.53	9.39
7	3	1.52	1.87	2.22	2.97	3.71	4.30	5.04	6.80	11.20	16.07	25.31
	4	1.59	1.88	2.19	2.86	3.47	3.94	4.49	5.69	8.39	10.80	14.80
	5	1.59	1.86	2.16	2.78	3.32	3.72	4.18	5.13	7.12	8.84	11.18
	6	1.57	1.84	2.14	2.72	3.21	3.57	3.96	4.76	6.33	7.61	9.40
	7	1.58	1.85	2.14	2.68	3.14	3.45	3.79	4.47	5.76	6.73	8.19
8	3	1.53	1.90	2.24	2.96	3.67	4.22	4.95	6.61	11.02	15.76	24.57
	4	1.63	1.91	2.22	2.87	3.46	3.91	4.44	5.60	8.19	10.74	15.22
	5	1.63	1.90	2.20	2.79	3.32	3.71	4.18	5.09	7.07	8.78	11.57
	6	1.62	1.89	2.18	2.75	3.24	3.58	3.99	4.77	6.35	7.67	9.43
	7	1.62	1.89	2.17	2.71	3.16	3.48	3.83	4.52	5.83	6.91	8.38
	8	1.62	1.89	2.17	2.69	3.10	3.38	3.71	4.31	5.44	6.29	7.50
9	3	1.55	1.93	2.26	2.95	3.63	4.18	4.85	6.45	10.71	15.33	23.80
	4	1.67	1.96	2.25	2.88	3.44	3.86	4.40	5.50	8.02	10.40	14.41
	5	1.69	1.95	2.23	2.80	3.33	3.70	4.13	5.07	6.90	8.59	11.05
	6	1.68	1.95	2.22	2.76	3.24	3.58	3.98	4.77	6.27	7.51	9.46
	7	1.67	1.94	2.21	2.73	3.18	3.50	3.85	4.53	5.86	6.91	8.40
	8	1.68	1.94	2.20	2.70	3.12	3.41	3.73	4.36	5.53	6.39	7.73
	9	1.69	1.95	2.19	2.68	3.08	3.35	3.63	4.19	5.22	6.00	7.09

TABLE B-4 - (Continued)

Percentiles of the Distribution of $V_{.95}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
10	3	1.51	1.91	2.26	2.91	3.56	4.06	4.70	6.23	10.24	14.50	23.00
	4	1.70	1.98	2.27	2.86	3.39	3.81	4.29	5.42	7.81	10.12	13.69
	5	1.72	1.97	2.24	2.80	3.31	3.65	4.07	4.97	6.82	8.39	11.00
	6	1.71	1.97	2.23	2.76	3.22	3.55	3.93	4.70	6.24	7.50	9.42
	7	1.71	1.96	2.22	2.73	3.18	3.48	3.80	4.50	5.79	6.83	8.29
	8	1.70	1.96	2.21	2.71	3.13	3.41	3.72	4.33	5.52	6.40	7.61
	9	1.71	1.96	2.21	2.69	3.08	3.35	3.64	4.20	5.23	6.01	7.12
	10	1.72	1.96	2.21	2.67	3.04	3.29	3.56	4.08	4.98	5.67	6.65
11	3	1.48	1.93	2.25	2.88	3.49	3.98	4.59	6.07	9.89	14.11	22.60
	4	1.73	2.01	2.27	2.83	3.37	3.76	4.24	5.31	7.71	10.03	14.44
	5	1.75	2.01	2.26	2.78	3.27	3.62	4.03	4.89	6.72	8.34	10.93
	6	1.75	2.00	2.24	2.74	3.19	3.52	3.90	4.63	6.16	7.42	9.39
	7	1.75	1.99	2.23	2.72	3.14	3.45	3.79	4.45	5.79	6.83	8.42
	8	1.75	1.99	2.22	2.70	3.10	3.39	3.70	4.31	5.46	6.38	7.65
	9	1.75	1.98	2.22	2.68	3.07	3.34	3.63	4.21	5.23	6.04	7.23
	10	1.76	2.00	2.22	2.67	3.04	3.28	3.56	4.09	5.03	5.75	6.73
	11	1.77	2.00	2.22	2.66	3.01	3.25	3.50	3.98	4.85	5.49	6.35
12	3	1.42	1.91	2.26	2.87	3.47	3.91	4.50	5.85	9.41	13.40	21.39
	4	1.74	2.02	2.28	2.84	3.34	3.72	4.18	5.16	7.42	9.56	13.27
	5	1.79	2.03	2.28	2.78	3.26	3.61	3.99	4.82	6.54	8.08	10.40
	6	1.79	2.02	2.27	2.75	3.20	3.52	3.86	4.56	5.97	7.22	9.00
	7	1.79	2.02	2.26	2.73	3.14	3.43	3.76	4.40	5.63	6.66	8.08
	8	1.79	2.01	2.25	2.71	3.11	3.38	3.68	4.26	5.36	6.27	7.49
	9	1.78	2.00	2.24	2.69	3.07	3.33	3.62	4.16	5.16	5.95	7.06
	10	1.77	2.01	2.23	2.67	3.04	3.29	3.56	4.07	4.99	5.67	6.63
	11	1.78	2.01	2.24	2.67	3.02	3.25	3.51	3.98	4.84	5.47	6.29
	12	1.80	2.02	2.24	2.66	3.00	3.22	3.46	3.91	4.68	5.26	6.00
13	3	1.44	1.92	2.27	2.88	3.45	3.90	4.47	5.84	9.23	13.11	20.76
	4	1.78	2.05	2.31	2.84	3.35	3.73	4.18	5.18	7.38	9.47	13.09
	5	1.82	2.06	2.30	2.81	3.26	3.60	3.98	4.80	6.57	8.04	10.25
	6	1.82	2.05	2.29	2.78	3.21	3.50	3.86	4.58	6.03	7.24	8.89
	7	1.82	2.05	2.28	2.75	3.15	3.44	3.75	4.41	5.65	6.68	8.10
	8	1.81	2.04	2.27	2.74	3.12	3.39	3.69	4.28	5.40	6.29	7.43
	9	1.81	2.04	2.27	2.72	3.10	3.35	3.63	4.16	5.17	6.00	6.99
	10	1.81	2.04	2.27	2.70	3.07	3.31	3.58	4.09	5.01	5.70	6.66
	11	1.82	2.04	2.27	2.70	3.05	3.27	3.53	4.02	4.87	5.50	6.36
	12	1.82	2.04	2.27	2.69	3.02	3.24	3.48	3.95	4.73	5.30	6.12
	13	1.83	2.05	2.27	2.68	3.00	3.21	3.43	3.87	4.61	5.12	5.80

TABLE B-4 - (Continued)

Percentiles of the Distribution of $V_{.95}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
14	3	1.39	1.92	2.26	2.84	3.39	3.83	4.36	5.64	8.84	12.73	19.14
	4	1.77	2.06	2.31	2.82	3.30	3.67	4.09	5.06	7.18	9.10	12.45
	5	1.84	2.08	2.31	2.79	3.22	3.55	3.93	4.72	6.38	7.82	9.93
	6	1.85	2.08	2.30	2.76	3.17	3.46	3.81	4.52	5.91	7.07	8.71
	7	1.85	2.07	2.29	2.73	3.12	3.40	3.72	4.38	5.58	6.53	7.78
	8	1.85	2.07	2.28	2.72	3.09	3.36	3.66	4.25	5.35	6.16	7.30
	9	1.84	2.07	2.28	2.71	3.07	3.32	3.59	4.15	5.14	5.88	6.92
	10	1.85	2.06	2.27	2.70	3.05	3.29	3.56	4.07	4.98	5.65	6.49
	11	1.85	2.07	2.28	2.69	3.03	3.26	3.51	4.00	4.86	5.48	6.26
	12	1.85	2.07	2.28	2.68	3.01	3.23	3.47	3.93	4.73	5.31	6.06
	13	1.85	2.07	2.29	2.68	2.99	3.21	3.44	3.87	4.61	5.14	5.80
	14	1.86	2.08	2.28	2.66	2.98	3.18	3.41	3.81	4.48	4.97	5.60
15	3	1.24	1.88	2.24	2.84	3.38	3.80	4.35	5.56	8.75	12.22	18.38
	4	1.77	2.06	2.31	2.83	3.30	3.65	4.07	4.99	7.00	8.90	11.93
	5	1.84	2.09	2.32	2.80	3.23	3.55	3.92	4.69	6.25	7.64	9.79
	6	1.87	2.09	2.31	2.79	3.18	3.47	3.80	4.48	5.79	6.91	8.55
	7	1.87	2.08	2.30	2.76	3.14	3.41	3.73	4.34	5.50	6.41	7.90
	8	1.86	2.08	2.29	2.74	3.11	3.36	3.66	4.23	5.29	6.10	7.26
	9	1.86	2.07	2.29	2.73	3.09	3.33	3.60	4.14	5.11	5.81	6.87
	10	1.85	2.07	2.28	2.72	3.06	3.30	3.56	4.08	4.96	5.61	6.50
	11	1.85	2.07	2.28	2.70	3.04	3.27	3.52	3.99	4.84	5.43	6.22
	12	1.86	2.07	2.28	2.70	3.02	3.25	3.48	3.94	4.73	5.31	6.00
	13	1.87	2.07	2.28	2.69	3.01	3.22	3.45	3.88	4.63	5.17	5.88
	14	1.87	2.07	2.28	2.68	3.00	3.20	3.41	3.83	4.53	5.02	5.66
	15	1.89	2.08	2.28	2.68	2.98	3.18	3.39	3.77	4.43	4.88	5.46
16	3	1.13	1.85	2.25	2.84	3.38	3.78	4.30	5.47	8.49	11.98	18.76
	4	1.77	2.06	2.33	2.83	3.30	3.64	4.06	4.95	7.00	8.88	12.30
	5	1.87	2.11	2.34	2.81	3.24	3.54	3.89	4.67	6.28	7.67	9.72
	6	1.89	2.11	2.33	2.79	3.19	3.48	3.81	4.47	5.80	6.92	8.63
	7	1.89	2.10	2.33	2.77	3.15	3.42	3.72	4.33	5.51	6.47	7.98
	8	1.89	2.10	2.32	2.75	3.12	3.38	3.66	4.22	5.28	6.12	7.27
	9	1.88	2.09	2.31	2.74	3.10	3.34	3.62	4.13	5.11	5.84	6.92
	10	1.88	2.09	2.31	2.72	3.07	3.32	3.57	4.07	4.95	5.60	6.60
	11	1.89	2.09	2.30	2.71	3.05	3.28	3.54	4.00	4.85	5.44	6.30
	12	1.88	2.09	2.31	2.71	3.03	3.25	3.50	3.94	4.71	5.30	6.06
	13	1.89	2.10	2.31	2.70	3.02	3.23	3.47	3.89	4.60	5.14	5.87
	14	1.90	2.10	2.31	2.69	3.00	3.21	3.43	3.84	4.52	5.02	5.68
	15	1.90	2.11	2.31	2.69	2.99	3.18	3.40	3.79	4.44	4.91	5.53
	16	1.92	2.12	2.33	2.69	2.98	3.16	3.36	3.74	4.35	4.80	5.35

TABLE B-4 - (Continued)

Percentiles of the Distribution of $V_{.95}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
17	3	1.07	1.84	2.22	2.80	3.29	3.67	4.15	5.26	8.19	11.24	17.44
	4	1.76	2.08	2.32	2.80	3.24	3.57	3.95	4.81	6.65	8.45	11.48
	5	1.86	2.11	2.33	2.79	3.19	3.48	3.83	4.57	5.98	7.34	9.21
	6	1.90	2.11	2.33	2.77	3.15	3.42	3.75	4.40	5.61	6.67	8.19
	7	1.90	2.11	2.32	2.76	3.11	3.38	3.67	4.26	5.39	6.26	7.56
	8	1.90	2.11	2.32	2.74	3.09	3.34	3.61	4.15	5.17	5.95	7.05
	9	1.89	2.10	2.31	2.73	3.07	3.30	3.56	4.07	5.01	5.72	6.70
	10	1.89	2.10	2.31	2.71	3.05	3.28	3.52	4.00	4.89	5.53	6.37
	11	1.89	2.10	2.31	2.71	3.03	3.25	3.48	3.94	4.77	5.35	6.16
	12	1.89	2.11	2.31	2.70	3.01	3.23	3.45	3.89	4.67	5.23	5.92
	13	1.89	2.11	2.31	2.69	3.00	3.20	3.42	3.84	4.58	5.10	5.79
	14	1.90	2.12	2.31	2.69	2.99	3.19	3.40	3.80	4.51	5.00	5.61
	15	1.90	2.11	2.31	2.68	2.98	3.17	3.38	3.77	4.43	4.88	5.44
	16	1.90	2.12	2.31	2.67	2.97	3.15	3.35	3.72	4.34	4.79	5.30
	17	1.92	2.12	2.32	2.67	2.96	3.14	3.33	3.68	4.27	4.68	5.19
18	3	1.11	1.83	2.23	2.80	3.29	3.66	4.12	5.23	7.96	11.18	17.89
	4	1.81	2.09	2.34	2.82	3.25	3.56	3.95	4.79	6.66	8.46	11.66
	5	1.90	2.14	2.36	2.80	3.20	3.49	3.83	4.56	6.00	7.32	9.43
	6	1.93	2.14	2.36	2.78	3.16	3.43	3.75	4.40	5.63	6.61	8.27
	7	1.94	2.14	2.35	2.77	3.13	3.38	3.68	4.25	5.39	6.23	7.45
	8	1.94	2.14	2.35	2.75	3.10	3.35	3.63	4.15	5.17	5.95	6.97
	9	1.94	2.14	2.34	2.74	3.08	3.32	3.58	4.09	5.03	5.72	6.71
	10	1.93	2.13	2.34	2.73	3.06	3.29	3.54	4.03	4.88	5.51	6.36
	11	1.93	2.13	2.34	2.73	3.04	3.26	3.51	3.98	4.79	5.35	6.15
	12	1.93	2.14	2.34	2.72	3.03	3.24	3.48	3.92	4.68	5.23	5.96
	13	1.93	2.14	2.33	2.71	3.01	3.22	3.45	3.86	4.60	5.12	5.75
	14	1.93	2.13	2.33	2.70	3.01	3.21	3.42	3.83	4.52	5.01	5.60
	15	1.94	2.14	2.34	2.70	3.00	3.19	3.40	3.79	4.44	4.88	5.46
	16	1.94	2.14	2.34	2.70	2.99	3.18	3.38	3.75	4.37	4.83	5.36
	17	1.95	2.15	2.34	2.69	2.97	3.16	3.35	3.71	4.31	4.74	5.22
	18	1.97	2.17	2.35	2.69	2.97	3.15	3.33	3.68	4.24	4.65	5.11
19	3	0.96	1.73	2.19	2.79	3.27	3.67	4.08	5.16	7.80	10.74	16.87
	4	1.76	2.08	2.34	2.81	3.24	3.54	3.92	4.72	6.52	8.23	11.18
	5	1.90	2.14	2.36	2.80	3.19	3.47	3.79	4.50	5.95	7.23	9.20
	6	1.94	2.15	2.36	2.78	3.15	3.42	3.72	4.35	5.56	6.60	8.07
	7	1.95	2.16	2.35	2.76	3.13	3.37	3.65	4.23	5.31	6.16	7.45
	8	1.95	2.15	2.35	2.75	3.10	3.34	3.60	4.14	5.14	5.87	6.94
	9	1.95	2.15	2.35	2.74	3.07	3.31	3.56	4.05	4.98	5.70	6.58
	10	1.95	2.14	2.34	2.73	3.05	3.27	3.52	4.00	4.84	5.49	6.35
	11	1.95	2.14	2.34	2.72	3.03	3.26	3.50	3.95	4.73	5.33	6.17
	12	1.95	2.14	2.34	2.72	3.02	3.24	3.47	3.89	4.65	5.22	5.97
	13	1.95	2.14	2.34	2.71	3.01	3.22	3.44	3.86	4.55	5.10	5.80
	14	1.95	2.14	2.33	2.70	3.00	3.20	3.41	3.81	4.49	4.98	5.61
	15	1.95	2.14	2.33	2.70	2.99	3.18	3.39	3.78	4.43	4.88	5.51
	16	1.95	2.15	2.33	2.70	2.98	3.17	3.37	3.74	4.38	4.81	5.35
	17	1.95	2.15	2.34	2.70	2.98	3.16	3.35	3.71	4.31	4.74	5.25
	18	1.96	2.16	2.35	2.70	2.97	3.14	3.33	3.68	4.26	4.65	5.15
	19	1.98	2.17	2.35	2.69	2.96	3.13	3.31	3.64	4.20	4.57	5.02

TABLE B-4 - (Continued)

Percentiles of the Distribution of $V_{.95}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
20	3	0.94	1.75	2.19	2.78	3.24	3.60	4.02	5.00	7.72	10.78	16.96
	4	1.79	2.09	2.34	2.81	3.21	3.52	3.88	4.66	6.49	8.16	11.03
	5	1.92	2.15	2.36	2.80	3.18	3.46	3.79	4.45	5.85	7.09	9.12
	6	1.94	2.17	2.37	2.79	3.15	3.40	3.71	4.31	5.48	6.49	8.11
	7	1.95	2.17	2.36	2.77	3.12	3.37	3.64	4.19	5.25	6.11	7.42
	8	1.96	2.17	2.36	2.76	3.09	3.32	3.59	4.09	5.06	5.88	6.97
	9	1.96	2.16	2.35	2.75	3.07	3.29	3.55	4.02	4.92	5.62	6.57
	10	1.95	2.16	2.35	2.74	3.05	3.27	3.51	3.97	4.79	5.43	6.30
	11	1.95	2.16	2.35	2.73	3.04	3.24	3.49	3.91	4.70	5.29	6.09
	12	1.95	2.16	2.35	2.72	3.02	3.22	3.45	3.87	4.61	5.18	5.92
	13	1.96	2.16	2.35	2.72	3.01	3.21	3.42	3.83	4.53	5.07	5.79
	14	1.96	2.16	2.34	2.71	3.00	3.19	3.40	3.79	4.46	4.96	5.63
	15	1.96	2.16	2.35	2.70	2.99	3.17	3.38	3.76	4.41	4.86	5.49
	16	1.97	2.16	2.35	2.70	2.98	3.16	3.36	3.73	4.35	4.78	5.37
	17	1.98	2.17	2.35	2.70	2.97	3.15	3.34	3.70	4.29	4.70	5.23
	18	1.98	2.18	2.35	2.69	2.96	3.14	3.33	3.67	4.23	4.64	5.14
	19	1.99	2.18	2.36	2.69	2.96	3.13	3.31	3.63	4.18	4.57	5.06
	20	2.00	2.19	2.36	2.70	2.95	3.12	3.29	3.60	4.12	4.49	4.95
21	3	0.74	1.67	2.16	2.75	3.21	3.56	3.97	4.97	7.52	10.37	15.72
	4	1.72	2.08	2.33	2.79	3.20	3.49	3.85	4.62	6.33	7.91	10.58
	5	1.92	2.15	2.37	2.79	3.17	3.44	3.76	4.41	5.82	7.02	8.86
	6	1.97	2.16	2.37	2.77	3.13	3.39	3.69	4.27	5.46	6.44	7.87
	7	1.98	2.17	2.37	2.76	3.11	3.35	3.63	4.17	5.22	6.05	7.22
	8	1.98	2.17	2.36	2.75	3.08	3.32	3.58	4.09	5.08	5.60	6.79
	9	1.98	2.16	2.36	2.75	3.07	3.29	3.54	4.02	4.93	5.58	6.51
	10	1.98	2.16	2.36	2.74	3.05	3.27	3.51	3.96	4.81	5.43	6.26
	11	1.98	2.16	2.36	2.73	3.04	3.25	3.48	3.92	4.71	5.28	6.07
	12	1.98	2.16	2.35	2.72	3.02	3.23	3.46	3.88	4.63	5.17	5.89
	13	1.98	2.16	2.35	2.72	3.01	3.21	3.43	3.84	4.56	5.05	5.73
	14	1.98	2.16	2.35	2.71	3.00	3.19	3.41	3.81	4.49	4.98	5.64
	15	1.98	2.16	2.35	2.70	2.99	3.17	3.39	3.77	4.42	4.89	5.49
	16	1.99	2.17	2.35	2.70	2.98	3.16	3.36	3.74	4.37	4.78	5.34
	17	2.00	2.17	2.35	2.70	2.98	3.15	3.35	3.71	4.30	4.71	5.24
	18	2.00	2.18	2.36	2.69	2.97	3.14	3.33	3.68	4.26	4.64	5.15
	19	2.01	2.18	2.36	2.69	2.96	3.13	3.32	3.65	4.20	4.59	5.08
	20	2.01	2.19	2.36	2.69	2.95	3.12	3.30	3.62	4.15	4.52	4.98
	21	2.03	2.20	2.37	2.69	2.95	3.11	3.28	3.59	4.09	4.44	4.87

TABLE B-4 - (Continued)

Percentiles of the Distribution of $V_{.95}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
22	3	0.58	1.60	2.15	2.74	3.18	3.62	3.93	4.86	7.32	9.93	15.15
	4	1.71	2.07	2.33	2.78	3.18	3.47	3.82	4.55	6.26	7.80	10.66
	5	1.92	2.17	2.37	2.78	3.15	3.42	3.73	4.38	5.75	6.85	8.68
	6	1.98	2.19	2.38	2.77	3.12	3.37	3.66	4.25	5.41	6.34	7.71
	7	1.99	2.19	2.38	2.76	3.10	3.34	3.61	4.16	5.22	6.03	7.20
	8	1.99	2.19	2.37	2.75	3.08	3.31	3.57	4.08	5.03	5.75	6.77
	9	1.99	2.19	2.37	2.74	3.06	3.29	3.53	4.01	4.88	5.53	6.44
	10	1.99	2.19	2.37	2.73	3.04	3.26	3.50	3.94	4.78	5.38	6.16
	11	1.99	2.18	2.36	2.72	3.03	3.24	3.47	3.91	4.67	5.24	5.98
	12	1.99	2.19	2.36	2.72	3.02	3.21	3.45	3.87	4.58	5.14	5.81
	13	2.00	2.19	2.36	2.71	3.00	3.20	3.42	3.82	4.52	5.04	5.66
	14	2.00	2.16	2.37	2.71	3.00	3.19	3.40	3.79	4.46	4.94	5.56
	15	2.00	2.19	2.36	2.71	2.99	3.17	3.38	3.75	4.39	4.84	5.44
	16	2.00	2.19	2.37	2.71	2.98	3.16	3.36	3.73	4.34	4.76	5.30
	17	2.00	2.19	2.36	2.71	2.97	3.15	3.34	3.70	4.29	4.69	5.23
	18	2.00	2.19	2.37	2.70	2.97	3.14	3.33	3.67	4.24	4.64	5.15
	19	2.02	2.19	2.37	2.70	2.96	3.14	3.32	3.65	4.20	4.59	5.06
	20	2.02	2.20	2.38	2.70	2.96	3.12	3.30	3.63	4.16	4.51	4.98
	21	2.02	2.21	2.38	2.70	2.95	3.11	3.29	3.60	4.11	4.45	4.89
	22	2.04	2.22	2.38	2.70	2.95	3.11	3.27	3.57	4.06	4.38	4.79
23	3	0.50	1.56	2.13	2.74	3.18	3.49	3.88	4.84	7.21	9.91	15.32
	4	1.69	2.07	2.33	2.79	3.18	3.46	3.80	4.54	6.20	7.84	10.66
	5	1.92	2.16	2.37	2.79	3.16	3.41	3.71	4.35	5.70	6.93	8.89
	6	1.99	2.19	2.38	2.79	3.13	3.37	3.64	4.22	5.33	6.34	7.78
	7	2.00	2.19	2.38	2.77	3.10	3.33	3.59	4.12	5.14	5.97	7.17
	8	2.00	2.19	2.38	2.76	3.08	3.31	3.55	4.04	4.98	5.68	6.76
	9	2.00	2.19	2.38	2.75	3.07	3.28	3.51	3.98	4.85	5.52	6.37
	10	2.00	2.19	2.37	2.74	3.05	3.26	3.48	3.92	4.73	5.33	6.16
	11	2.00	2.19	2.37	2.74	3.03	3.24	3.46	3.88	4.63	5.21	5.93
	12	2.00	2.18	2.37	2.73	3.02	3.22	3.43	3.84	4.55	5.08	5.78
	13	2.00	2.18	2.37	2.73	3.01	3.20	3.41	3.80	4.48	4.98	5.59
	14	2.00	2.18	2.37	2.72	3.00	3.18	3.39	3.77	4.45	4.91	5.52
	15	2.01	2.19	2.36	2.71	3.00	3.17	3.37	3.74	4.39	4.81	5.35
	16	2.01	2.19	2.36	2.71	2.99	3.16	3.35	3.72	4.33	4.75	5.31
	17	2.02	2.19	2.36	2.71	2.98	3.15	3.34	3.69	4.29	4.70	5.20
	18	2.02	2.19	2.37	2.71	2.97	3.14	3.33	3.67	4.24	4.63	5.14
	19	2.01	2.20	2.37	2.71	2.97	3.13	3.31	3.65	4.20	4.58	5.07
	20	2.02	2.20	2.37	2.70	2.96	3.12	3.30	3.62	4.15	4.53	4.99
	21	2.03	2.20	2.37	2.70	2.95	3.12	3.29	3.60	4.11	4.47	4.92
	22	2.04	2.21	2.38	2.70	2.95	3.10	3.28	3.58	4.07	4.42	4.82
	23	2.05	2.21	2.38	2.70	2.94	3.10	3.26	3.55	4.03	4.36	4.74

TABLE B-4 - (Concluded)

Percentiles of the Distribution of $\bar{V}_{.95}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
24	3	0.33	1.49	2.08	2.72	3.15	3.47	3.86	4.77	7.09	9.55	14.35
	4	1.66	2.05	2.33	2.79	3.15	3.44	3.78	4.50	6.13	7.65	10.28
	5	1.90	2.17	2.38	2.79	3.14	3.40	3.71	4.35	5.64	6.72	8.52
	6	1.98	2.20	2.39	2.79	3.12	3.37	3.66	4.22	5.34	6.24	7.68
	7	2.00	2.21	2.39	2.78	3.10	3.33	3.61	4.13	5.14	5.92	7.06
	8	2.00	2.21	2.38	2.77	3.08	3.30	3.56	4.05	4.95	5.69	6.67
	9	2.01	2.21	2.38	2.76	3.06	3.28	3.52	3.99	4.84	5.47	6.34
	10	2.01	2.20	2.38	2.75	3.05	3.25	3.48	3.93	4.73	5.33	6.13
	11	2.00	2.20	2.38	2.75	3.04	3.24	3.46	3.89	4.64	5.21	5.94
	12	2.00	2.19	2.37	2.74	3.03	3.22	3.44	3.84	4.57	5.10	5.74
	13	2.01	2.19	2.37	2.73	3.02	3.21	3.41	3.81	4.50	5.00	5.67
	14	2.00	2.19	2.37	2.73	3.01	3.20	3.40	3.77	4.43	4.89	5.48
	15	2.01	2.19	2.37	2.73	3.00	3.19	3.38	3.75	4.37	4.81	5.33
	16	2.01	2.19	2.37	2.73	2.99	3.17	3.37	3.72	4.32	4.74	5.27
	17	2.01	19	2.38	2.72	2.99	3.16	3.35	3.70	4.28	4.70	5.18
	18	2.01	20	2.38	2.72	2.98	3.15	3.34	3.68	4.23	4.63	5.10
	19	2.02	2.20	2.38	2.71	2.98	3.14	3.32	3.65	4.19	4.56	5.05
	20	2.03	2.21	2.38	2.71	2.97	3.13	3.31	3.63	4.15	4.52	4.96
	21	2.03	2.21	2.38	2.72	2.96	3.13	3.30	3.61	4.10	4.46	4.90
	22	2.03	2.22	2.39	2.71	2.95	3.11	3.28	3.59	4.07	4.41	4.84
	23	2.05	2.22	2.39	2.71	2.95	3.11	3.26	3.57	4.04	4.35	4.77
	24	2.05	2.23	2.40	2.71	2.95	3.10	3.25	3.54	3.99	4.30	4.70
25	3	0.19	1.44	2.06	2.72	3.15	3.45	3.82	4.68	6.85	9.30	14.21
	4	1.64	2.06	2.35	2.79	3.16	3.42	3.75	4.45	5.96	7.49	9.99
	5	1.94	2.18	2.40	2.80	3.14	3.39	3.69	4.29	5.49	6.65	8.40
	6	2.01	2.21	2.41	2.80	3.13	3.35	3.63	4.19	5.25	6.14	7.48
	7	2.03	2.22	2.41	2.78	3.11	3.33	3.59	4.11	5.07	5.85	6.98
	8	2.04	2.22	2.41	2.77	3.09	3.30	3.55	4.03	4.90	5.60	6.60
	9	2.04	2.22	2.41	2.76	3.08	3.28	3.51	3.96	4.79	5.45	6.39
	10	2.04	2.22	2.40	2.76	3.06	3.27	3.48	3.91	4.69	5.27	6.10
	11	2.03	2.22	2.40	2.75	3.05	3.25	3.46	3.87	4.60	5.16	5.88
	12	2.04	2.22	2.40	2.74	3.04	3.23	3.44	3.83	4.53	5.05	5.79
	13	2.03	2.21	2.39	2.73	3.02	3.21	3.42	3.81	4.48	4.96	5.64
	14	2.03	2.22	2.39	2.73	3.01	3.20	3.40	3.77	4.42	4.88	5.52
	15	2.03	2.22	2.39	2.73	3.00	3.18	3.39	3.75	4.35	4.80	5.40
	16	2.03	2.22	2.39	2.73	3.00	3.17	3.37	3.71	4.31	4.73	5.30
	17	2.03	2.22	2.39	2.73	2.99	3.16	3.35	3.69	4.29	4.68	5.17
	18	2.04	2.22	2.39	2.72	2.98	3.15	3.33	3.67	4.23	4.62	5.11
	19	2.05	2.22	2.39	2.72	2.97	3.14	3.32	3.65	4.19	4.56	5.01
	20	2.05	2.22	2.40	2.72	2.97	3.13	3.31	3.62	4.16	4.51	4.93
	21	2.05	2.23	2.40	2.72	2.96	3.12	3.29	3.61	4.12	4.45	4.88
	22	2.05	2.23	2.40	2.72	2.96	3.11	3.28	3.59	4.09	4.42	4.83
	23	2.07	2.24	2.41	2.71	2.95	3.11	3.27	3.57	4.05	4.39	4.77
	24	2.07	2.25	2.41	2.71	2.95	3.10	3.26	3.55	4.02	4.34	4.74
	25	2.08	2.26	2.42	2.71	2.94	3.09	3.25	3.53	3.99	4.28	4.67

TABLE B-5

Percentiles of the Distribution of $V_{.99}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
3	3	2.24	2.75	3.32	4.63	5.93	6.96	8.19	11.05	18.15	26.71	42.07
4	3	2.38	2.87	3.44	4.74	6.02	7.07	8.38	11.31	19.38	27.97	43.72
	4	2.37	2.82	3.31	4.38	5.34	6.09	6.91	8.74	12.79	16.62	23.12
5	3	2.45	2.95	3.47	4.80	6.09	7.14	8.48	11.55	19.73	28.71	46.68
	4	2.43	2.86	3.37	4.44	5.44	6.20	7.09	9.00	13.31	17.41	24.31
	5	2.45	2.87	3.32	4.28	5.11	5.71	6.39	7.81	10.75	13.23	16.87
6	3	2.55	3.07	3.55	4.80	6.08	7.11	8.39	11.42	19.28	28.02	45.73
	4	2.52	2.96	3.45	4.49	5.50	6.25	7.16	9.01	13.49	17.54	24.27
	5	2.52	2.94	3.38	4.33	5.18	5.79	6.52	7.95	10.96	13.67	17.96
	6	2.56	2.94	3.39	4.25	4.96	5.49	6.08	7.26	9.49	11.41	14.37
7	3	2.64	3.11	3.61	4.81	6.07	7.04	8.37	11.40	19.27	27.76	44.93
	4	2.61	3.04	3.49	4.52	5.48	6.25	7.14	9.08	13.53	17.47	23.96
	5	2.59	3.00	3.44	4.37	5.19	5.80	6.51	8.02	11.20	13.91	17.78
	6	2.61	3.01	3.41	4.26	5.00	5.53	6.14	7.32	9.75	11.74	14.36
	7	2.65	3.02	3.43	4.21	4.87	5.32	5.82	6.83	8.75	10.20	12.38
8	3	2.67	3.12	3.65	4.82	6.01	7.03	8.27	11.28	19.24	27.78	44.29
	4	2.66	3.07	3.53	4.54	5.51	6.23	7.12	9.04	13.42	17.64	25.03
	5	2.64	3.05	3.47	4.39	5.20	5.83	6.55	9.03	11.12	13.92	18.42
	6	2.64	3.04	3.45	4.30	5.04	5.57	6.17	7.41	9.82	11.87	14.78
	7	2.67	3.06	3.47	4.24	4.90	5.38	5.90	6.95	8.92	10.52	12.89
	8	2.71	3.08	3.47	4.20	4.79	5.21	5.69	6.58	8.27	9.52	11.35
9	3	2.75	3.20	3.66	4.81	6.02	6.97	8.17	11.14	19.00	27.91	43.02
	4	2.72	3.12	3.56	4.56	5.49	6.21	7.10	8.99	13.28	17.57	24.34
	5	2.71	3.10	3.52	4.40	5.23	5.83	6.53	8.05	10.98	13.78	17.99
	6	2.72	3.10	3.52	4.32	5.06	5.59	6.20	7.43	9.82	11.78	14.84
	7	2.71	3.10	3.51	4.27	4.96	5.42	5.95	6.99	9.06	10.66	12.96
	8	2.75	3.12	3.50	4.24	4.84	5.27	5.74	6.68	8.44	9.75	11.85
	9	2.79	3.17	3.50	4.20	4.78	5.16	5.59	6.41	7.90	9.04	10.71

TABLE B-5 - (Continued)

Percentiles of the Distribution of $V_{.99}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
10	3	2.75	3.20	3.68	4.77	5.91	6.86	8.10	10.93	18.61	27.05	43.71
	4	2.74	3.16	3.60	4.53	5.45	6.15	7.00	8.87	13.12	17.17	23.48
	5	2.75	3.13	3.54	4.41	5.22	5.79	6.47	7.94	11.05	13.70	17.82
	6	2.75	3.11	3.51	4.33	5.04	5.56	6.15	7.38	9.81	11.86	15.01
	7	2.77	3.12	3.50	4.28	4.95	5.42	5.97	6.99	8.99	10.66	13.03
	8	2.77	3.15	3.50	4.23	4.86	5.29	5.75	6.68	8.49	9.88	11.69
	9	2.81	3.16	3.51	4.21	4.78	5.17	5.60	6.43	7.97	9.17	10.82
	10	2.84	3.19	3.53	4.18	4.72	5.07	5.48	6.23	7.57	8.57	10.03
11	3	2.79	3.22	3.67	4.75	5.87	6.79	7.98	10.78	18.19	26.45	43.82
	4	2.79	3.16	3.58	4.53	5.43	6.13	6.96	8.81	13.02	17.25	24.97
	5	2.79	3.15	3.55	4.39	5.19	5.77	6.44	7.88	10.99	13.66	18.13
	6	2.77	3.14	3.52	4.30	5.02	5.53	6.13	7.33	9.79	11.86	15.02
	7	2.78	3.16	3.51	4.26	4.92	5.39	5.92	6.95	9.03	10.72	13.30
	8	2.81	3.16	3.51	4.23	4.83	5.27	5.74	6.69	8.43	9.86	11.88
	9	2.84	3.17	3.51	4.20	4.77	5.16	5.61	6.49	8.02	9.24	11.08
	10	2.86	3.20	3.53	4.18	4.71	5.09	5.49	6.27	7.67	8.73	10.20
	11	2.88	3.24	3.55	4.17	4.66	5.01	5.38	6.09	7.36	8.31	9.57
12	3	2.88	3.28	3.72	4.77	5.85	6.76	7.88	10.59	17.59	25.73	41.44
	4	2.85	3.22	3.62	4.52	5.41	6.08	6.81	8.67	12.72	16.60	23.41
	5	2.82	3.19	3.58	4.39	5.17	5.75	6.41	7.79	10.68	13.37	17.35
	6	2.82	3.19	3.55	4.31	5.03	5.54	6.12	7.26	9.61	11.60	14.59
	7	2.81	3.18	3.54	4.27	4.92	5.39	5.89	6.89	8.85	10.55	12.65
	8	2.84	3.18	3.54	4.24	4.83	5.25	5.73	6.63	8.34	9.75	11.64
	9	2.86	3.18	3.53	4.21	4.77	5.16	5.60	6.44	7.98	9.18	10.90
	10	2.86	3.20	3.54	4.18	4.73	5.10	5.50	6.26	7.66	8.68	10.17
	11	2.90	3.23	3.55	4.17	4.69	5.03	5.41	6.12	7.37	8.31	9.53
	12	2.94	3.27	3.57	4.17	4.64	4.97	5.32	5.97	7.09	7.96	9.03
13	3	2.88	3.30	3.74	4.78	5.87	6.76	7.89	10.68	17.58	25.37	40.28
	4	2.89	3.25	3.66	4.56	5.46	6.11	6.94	8.75	12.81	16.56	23.73
	5	2.87	3.22	3.60	4.43	5.20	5.78	6.43	7.81	10.89	13.47	17.15
	6	2.89	3.21	3.57	4.35	5.05	5.54	6.14	7.32	9.71	11.72	14.47
	7	2.89	3.21	3.57	4.30	4.94	5.38	5.92	6.94	8.97	10.61	13.04
	8	2.89	3.22	3.57	4.28	4.86	5.29	5.75	6.69	8.46	9.88	11.74
	9	2.91	3.23	3.56	4.25	4.81	5.21	5.64	6.47	8.02	9.27	10.88
	10	2.91	3.24	3.57	4.23	4.78	5.14	5.54	6.31	7.72	8.80	10.25
	11	2.94	3.25	3.58	4.22	4.73	5.07	5.44	6.18	7.46	8.42	9.65
	12	2.97	3.27	3.60	4.21	4.68	5.00	5.37	6.05	7.21	8.03	9.27
	13	3.00	3.30	3.61	4.20	4.66	4.95	5.28	5.91	6.99	7.75	8.76

TABLE B-5 - (Continued)

Percentiles of the Distribution of $V_{.99}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
14	3	2.90	3.30	3.75	4.75	5.80	6.63	7.76	10.37	17.23	25.09	39.16
	4	2.91	3.28	3.68	4.52	5.39	6.05	6.83	8.61	12.48	16.19	22.41
	5	2.91	3.25	3.62	4.40	5.14	5.71	6.38	7.77	10.64	13.22	16.89
	6	2.88	3.24	3.59	4.33	5.00	5.50	6.07	7.25	9.59	11.55	14.29
	7	2.91	3.23	3.58	4.28	4.90	5.35	5.86	6.92	8.88	10.47	12.51
	8	2.91	3.24	3.58	4.25	4.83	5.25	5.72	6.66	8.40	9.71	11.54
	9	2.93	3.25	3.58	4.23	4.79	5.16	5.59	6.45	8.00	9.17	10.81
	10	2.95	3.26	3.58	4.22	4.74	5.11	5.52	6.31	7.71	8.72	10.06
	11	2.95	3.28	3.60	4.20	4.71	5.06	5.44	6.17	7.46	8.42	9.61
	12	2.98	3.31	3.61	4.20	4.67	5.00	5.36	6.04	7.25	8.11	9.25
	13	3.00	3.31	3.62	4.19	4.65	4.96	5.30	5.93	7.03	7.82	8.81
	14	3.02	3.34	3.63	4.17	4.62	4.92	5.24	5.82	6.81	7.49	8.45
15	3	2.96	3.34	3.77	4.78	5.83	6.72	7.82	10.45	17.15	24.54	38.14
	4	2.94	3.29	3.69	4.55	5.40	6.05	6.82	8.54	12.41	15.96	21.82
	5	2.93	3.28	3.64	4.42	5.18	5.72	6.36	7.72	10.56	12.94	16.99
	6	2.93	3.26	3.61	4.37	5.03	5.52	6.09	7.23	9.45	11.38	14.24
	7	2.93	3.25	3.59	4.31	4.92	5.37	5.90	6.89	8.81	10.31	12.84
	8	2.93	3.25	3.59	4.27	4.86	5.27	5.73	6.64	8.36	9.64	11.58
	9	2.95	3.26	3.58	4.26	4.81	5.19	5.62	6.48	8.00	9.11	10.79
	10	2.94	3.27	3.59	4.24	4.77	5.13	5.54	6.33	7.71	8.72	10.06
	11	2.95	3.26	3.59	4.21	4.73	5.08	5.46	6.18	7.48	8.37	9.61
	12	2.98	3.29	3.61	4.21	4.70	5.04	5.39	6.07	7.29	8.17	9.24
	13	3.01	3.30	3.61	4.20	4.68	4.99	5.33	5.96	7.09	7.89	8.92
	14	3.03	3.32	3.62	4.19	4.65	4.95	5.26	5.86	6.92	7.63	8.59
	15	3.07	3.35	3.62	4.19	4.63	4.91	5.21	5.77	6.70	7.37	8.24
16	3	2.98	3.36	3.78	4.79	5.86	6.70	7.78	10.30	17.06	24.06	38.50
	4	2.96	3.32	3.70	4.57	5.41	6.04	6.82	8.52	12.49	16.24	23.03
	5	2.94	3.30	3.65	4.46	5.18	5.73	6.37	7.71	10.64	13.23	16.90
	6	2.93	3.29	3.63	4.37	5.05	5.53	6.11	7.23	9.53	11.53	14.38
	7	2.94	3.28	3.62	4.32	4.96	5.40	5.89	6.89	8.84	10.54	12.97
	8	2.95	3.28	3.62	4.30	4.88	5.30	5.75	6.65	8.39	9.75	11.59
	9	2.96	3.28	3.61	4.27	4.84	5.22	5.66	6.47	8.03	9.18	10.96
	10	2.97	3.28	3.62	4.25	4.79	5.16	5.55	6.32	7.71	8.75	10.29
	11	2.99	3.29	3.62	4.23	4.74	5.10	5.48	6.19	7.50	8.43	9.76
	12	2.99	3.31	3.63	4.23	4.71	5.04	5.41	6.09	7.27	8.17	9.32
	13	3.01	3.34	3.63	4.21	4.68	5.01	5.36	5.99	7.07	7.86	9.04
	14	3.05	3.35	3.65	4.21	4.66	4.96	5.30	5.90	6.93	7.68	8.67
	15	3.07	3.37	3.66	4.21	4.64	4.92	5.24	5.80	6.78	7.46	8.39
	16	3.12	3.39	3.69	4.20	4.62	4.88	5.17	5.72	6.60	7.26	8.10

TABLE B-5 - (Continued)

Percentiles of the Distribution of $V_{.99}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
17	3	2.97	3.37	3.76	4.71	5.69	6.50	7.56	10.06	16.55	23.57	37.77
	4	2.97	3.32	3.70	4.51	5.32	5.94	6.71	8.36	11.93	15.54	21.68
	5	2.96	3.30	3.66	4.42	5.12	5.65	6.27	7.62	10.23	12.58	16.34
	6	2.95	3.28	3.63	4.34	5.01	5.47	6.03	7.14	9.32	11.13	13.79
	7	2.96	3.29	3.63	4.30	4.90	5.34	5.83	6.83	8.70	10.18	12.33
	8	2.97	3.28	3.61	4.27	4.83	5.23	5.69	6.58	8.25	9.56	11.40
	9	2.97	3.29	3.61	4.25	4.79	5.17	5.58	6.39	7.91	9.07	10.65
	10	2.98	3.30	3.61	4.23	4.76	5.12	5.50	6.25	7.66	8.70	10.01
	11	2.99	3.31	3.62	4.22	4.71	5.06	5.42	6.13	7.42	8.33	9.63
	12	3.01	3.33	3.63	4.21	4.69	5.01	5.36	6.02	7.22	8.08	9.21
	13	3.03	3.34	3.63	4.21	4.66	4.97	5.30	5.93	7.05	7.86	8.92
	14	3.04	3.35	3.64	4.19	4.64	4.93	5.25	5.85	6.91	7.67	8.63
	15	3.05	3.37	3.66	4.19	4.62	4.91	5.21	5.79	6.77	7.45	8.29
	16	3.09	3.39	3.67	4.19	4.60	4.87	5.17	5.71	6.62	7.29	8.04
	17	3.12	3.41	3.68	4.18	4.58	4.84	5.12	5.63	6.48	7.11	7.84
18	3	3.03	3.38	3.78	4.72	5.69	6.50	7.53	9.97	16.20	23.42	38.75
	4	3.03	3.35	3.72	4.55	5.33	5.94	6.72	8.32	12.10	15.75	21.93
	5	3.01	3.35	3.68	4.44	5.14	5.67	6.29	7.61	10.29	12.75	16.76
	6	3.00	3.33	3.66	4.37	5.01	5.47	6.04	7.15	9.34	11.10	14.07
	7	3.00	3.33	3.65	4.33	4.93	5.35	5.86	6.84	8.80	10.17	12.25
	8	3.01	3.33	3.65	4.30	4.87	5.28	5.73	6.60	8.28	9.62	11.30
	9	3.03	3.33	3.64	4.27	4.82	5.19	5.61	6.46	7.97	9.12	10.71
	10	3.04	3.34	3.64	4.26	4.77	5.15	5.54	6.31	7.63	8.69	10.04
	11	3.04	3.34	3.64	4.25	4.74	5.08	5.47	6.20	7.45	8.34	9.69
	12	3.05	3.35	3.66	4.25	4.71	5.03	5.39	6.09	7.26	8.11	9.24
	13	3.06	3.37	3.67	4.23	4.68	4.99	5.34	5.99	7.10	7.92	8.84
	14	3.08	3.38	3.67	4.21	4.67	4.97	5.29	5.90	6.95	7.70	8.60
	15	3.11	3.39	3.68	4.21	4.65	4.94	5.25	5.82	6.82	7.50	8.32
	16	3.13	3.41	3.69	4.21	4.64	4.91	5.22	5.76	6.68	7.37	8.15
	17	3.16	3.43	3.70	4.20	4.61	4.88	5.17	5.69	6.57	7.19	7.94
	18	3.19	3.47	3.73	4.20	4.60	4.86	5.13	5.63	6.45	7.04	7.75
19	3	3.03	3.40	3.79	4.73	5.72	6.52	7.53	9.96	16.17	23.13	36.55
	4	3.05	3.37	3.73	4.55	5.35	5.95	6.70	8.27	11.98	15.31	21.70
	5	3.03	3.36	3.69	4.43	5.14	5.65	6.27	7.54	10.29	12.70	16.31
	6	3.02	3.34	3.66	4.36	5.01	5.47	6.01	7.10	9.28	11.15	13.95
	7	3.02	3.33	3.66	4.33	4.93	5.36	5.83	6.80	8.65	10.16	12.41
	8	3.02	3.33	3.64	4.29	4.87	5.25	5.69	6.58	8.25	9.52	11.35
	9	3.03	3.33	3.65	4.26	4.80	5.18	5.59	6.40	7.93	9.09	10.61
	10	3.03	3.34	3.64	4.25	4.76	5.11	5.52	6.26	7.63	8.67	10.08
	11	3.06	3.35	3.65	4.24	4.72	5.07	5.44	6.15	7.42	8.34	9.70
	12	3.07	3.37	3.65	4.23	4.70	5.03	5.39	6.05	7.22	8.13	9.34
	13	3.08	3.38	3.65	4.22	4.68	4.99	5.33	5.97	7.06	7.88	8.99
	14	3.09	3.38	3.66	4.21	4.66	4.96	5.28	5.89	6.93	7.69	8.63
	15	3.10	3.40	3.67	4.22	4.64	4.93	5.24	5.81	6.79	7.49	8.46
	16	3.13	3.42	3.68	4.21	4.63	4.90	5.20	5.76	6.70	7.35	8.19
	17	3.14	3.43	3.69	4.21	4.62	4.88	5.16	5.70	6.59	7.23	8.01
	18	3.17	3.44	3.71	4.21	4.60	4.86	5.14	5.64	6.49	7.08	7.84
	19	3.20	3.46	3.72	4.21	4.59	4.83	5.10	5.57	6.38	6.91	7.60

TABLE B-5 - (Continued)

Percentiles of the Distribution of $V_{.99}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
20	3	3.01	3.39	3.79	4.72	5.67	6.42	7.38	9.80	16.16	23.55	37.26
	4	3.02	3.37	3.73	4.54	5.32	5.93	6.66	8.21	11.97	15.38	21.61
	5	3.03	3.36	3.70	4.44	5.13	5.64	6.26	7.53	10.15	12.57	16.40
	6	3.02	3.34	3.67	4.38	5.01	5.46	6.01	7.08	9.17	11.05	14.08
	7	3.02	3.34	3.66	4.33	4.93	5.35	5.82	6.78	8.63	10.10	12.40
	8	3.03	3.35	3.66	4.30	4.85	5.25	5.68	6.53	8.18	9.55	11.47
	9	3.04	3.34	3.65	4.28	4.80	5.17	5.58	6.36	7.86	9.02	10.65
	10	3.04	3.35	3.65	4.26	4.76	5.12	5.50	6.24	7.59	8.61	10.02
	11	3.06	3.36	3.66	4.25	4.73	5.06	5.45	6.12	7.37	8.35	9.57
	12	3.08	3.38	3.66	4.24	4.70	5.03	5.38	6.03	7.20	8.08	9.28
	13	3.09	3.38	3.67	4.23	4.68	4.98	5.33	5.95	7.01	7.86	9.02
	14	3.11	3.40	3.67	4.22	4.65	4.95	5.26	5.87	6.89	7.66	8.68
	15	3.12	3.41	3.69	4.21	4.63	4.92	5.23	5.80	6.79	7.49	8.44
	16	3.14	3.42	3.70	4.21	4.62	4.89	5.20	5.74	6.67	7.34	8.23
	17	3.16	3.44	3.71	4.20	4.61	4.87	5.16	5.69	6.57	7.22	7.98
	18	3.18	3.46	3.72	4.20	4.60	4.86	5.13	5.63	6.47	7.07	7.82
	19	3.21	3.47	3.73	4.21	4.58	4.84	5.11	5.57	6.38	6.96	7.64
	20	3.24	3.49	3.74	4.21	4.57	4.81	5.06	5.52	6.25	6.80	7.48
21	3	3.05	3.40	3.79	4.70	5.66	6.42	7.38	9.68	16.03	22.96	36.33
	4	3.08	3.39	3.74	4.53	5.31	5.91	6.61	8.15	11.66	14.96	20.76
	5	3.05	3.37	3.69	4.44	5.13	5.64	6.23	7.46	10.16	12.46	16.03
	6	3.05	3.35	3.67	4.36	4.99	5.45	5.97	7.03	9.22	10.99	13.71
	7	3.05	3.35	3.66	4.32	4.92	5.33	5.82	6.75	8.57	10.02	12.10
	8	3.05	3.35	3.66	4.29	4.85	5.25	5.68	6.54	8.17	9.45	11.25
	9	3.06	3.35	3.65	4.28	4.80	5.16	5.58	6.38	7.91	8.97	10.54
	10	3.07	3.35	3.66	4.26	4.77	5.11	5.50	6.25	7.62	8.62	10.00
	11	3.08	3.36	3.66	4.25	4.74	5.07	5.44	6.13	7.44	8.34	9.64
	12	3.09	3.36	3.67	4.24	4.71	5.03	5.38	6.06	7.27	8.11	9.23
	13	3.10	3.38	3.68	4.23	4.68	4.99	5.33	5.98	7.10	7.88	8.94
	14	3.12	3.39	3.67	4.22	4.66	4.96	5.29	5.90	6.96	7.72	8.70
	15	3.14	3.40	3.68	4.21	4.64	4.93	5.25	5.83	6.83	7.55	8.49
	16	3.16	3.42	3.69	4.21	4.63	4.90	5.20	5.78	6.72	7.37	8.23
	17	3.17	3.44	3.69	4.20	4.62	4.89	5.17	5.72	6.61	7.24	8.04
	18	3.19	3.45	3.71	4.20	4.60	4.86	5.15	5.66	6.52	7.10	7.89
	19	3.21	3.47	3.72	4.21	4.60	4.85	5.12	5.60	6.42	7.00	7.72
	20	3.24	3.49	3.74	4.20	4.59	4.83	5.09	5.56	6.32	6.87	7.58
	21	3.27	3.51	3.75	4.21	4.57	4.31	5.06	5.51	6.22	6.73	7.42

TABLE B-5 - (Continued)

Percentiles of the Distribution of $V_{.99}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
22	3	3.06	3.42	3.80	4.67	5.60	6.38	7.33	9.61	15.80	22.00	34.81
	4	3.10	3.41	3.76	4.52	5.29	5.87	6.56	8.12	11.67	15.09	20.79
	5	3.09	3.38	3.71	4.42	5.09	5.60	6.19	7.42	10.06	12.25	16.00
	6	3.08	3.36	3.69	4.36	4.97	5.43	5.96	6.98	9.18	10.91	13.39
	7	3.08	3.36	3.67	4.32	4.90	5.31	5.79	6.74	8.65	10.07	12.18
	8	3.08	3.38	3.67	4.28	4.83	5.24	5.67	6.54	8.18	9.40	11.20
	9	3.09	3.38	3.68	4.26	4.79	5.17	5.57	6.37	7.85	8.97	10.51
	10	3.09	3.38	3.67	4.24	4.75	5.11	5.50	6.23	7.58	8.61	9.89
	11	3.10	3.38	3.67	4.23	4.72	5.07	5.44	6.14	7.36	8.29	9.53
	12	3.12	3.40	3.68	4.23	4.69	5.01	5.37	6.05	7.19	8.06	9.18
	13	3.14	3.42	3.68	4.22	4.67	4.98	5.32	5.95	7.05	7.85	8.87
	14	3.15	3.42	3.69	4.22	4.66	4.96	5.28	5.88	6.91	7.66	8.67
	15	3.16	3.43	3.70	4.22	4.64	4.94	5.25	5.81	6.79	7.48	8.43
	16	3.17	3.44	3.71	4.21	4.63	4.90	5.20	5.76	6.68	7.35	8.21
	17	3.18	3.45	3.71	4.22	4.61	4.88	5.17	5.70	6.60	7.19	8.03
	18	3.19	3.47	3.72	4.21	4.60	4.85	5.15	5.65	6.51	7.11	7.87
	19	3.21	3.47	3.73	4.21	4.60	4.85	5.12	5.61	6.44	7.00	7.72
	20	3.24	3.49	3.74	4.22	4.59	4.83	5.09	5.57	6.36	6.87	7.58
	21	3.25	3.51	3.75	4.22	4.58	4.82	5.07	5.53	6.27	6.77	7.43
	22	3.29	3.53	3.77	4.22	4.57	4.81	5.04	5.48	6.18	6.65	7.25
23	3	3.10	3.45	3.82	4.71	5.61	6.36	7.33	9.63	15.57	21.97	35.77
	4	3.12	3.43	3.77	4.53	5.31	5.87	6.57	8.18	11.82	15.36	21.34
	5	3.11	3.41	3.72	4.46	5.11	5.62	6.21	7.45	10.11	12.46	16.45
	6	3.10	3.39	3.69	4.39	5.00	5.44	5.96	7.00	9.07	10.93	13.64
	7	3.09	3.38	3.69	4.34	4.91	5.30	5.78	6.70	8.56	10.01	12.34
	8	3.09	3.38	3.68	4.31	4.85	5.23	5.64	6.49	8.13	9.40	11.19
	9	3.09	3.38	3.68	4.29	4.81	5.16	5.55	6.33	7.83	8.93	10.45
	10	3.09	3.38	3.68	4.27	4.77	5.11	5.47	6.20	7.56	8.55	9.95
	11	3.10	3.40	3.69	4.26	4.74	5.06	5.41	6.10	7.36	8.27	9.48
	12	3.11	3.40	3.69	4.25	4.72	5.02	5.36	6.01	7.16	8.02	9.08
	13	3.13	3.41	3.70	4.25	4.69	4.99	5.31	5.92	7.04	7.83	8.80
	14	3.14	3.42	3.70	4.23	4.67	4.96	5.27	5.86	6.92	7.66	8.63
	15	3.16	3.44	3.70	4.22	4.66	4.92	5.24	5.80	6.80	7.47	8.36
	16	3.16	3.44	3.70	4.23	4.64	4.91	5.20	5.75	6.68	7.35	8.16
	17	3.19	3.45	3.71	4.22	4.62	4.89	5.17	5.70	6.62	7.23	8.00
	18	3.20	3.47	3.72	4.22	4.61	4.87	5.15	5.65	6.53	7.10	7.90
	19	3.21	3.47	3.72	4.22	4.60	4.85	5.12	5.61	6.43	7.01	7.73
	20	3.23	3.48	3.73	4.22	4.59	4.83	5.09	5.57	6.34	6.92	7.57
	21	3.24	3.49	3.74	4.22	4.58	4.82	5.07	5.52	6.28	6.80	7.46
	22	3.26	3.51	3.76	4.22	4.57	4.80	5.05	5.50	6.22	6.72	7.32
	23	3.30	3.53	3.76	4.22	4.56	4.79	5.02	5.45	6.14	6.61	7.17

TABLE B-5 - (Concluded)

Percentiles of the Distribution of $V_{.99}$

n	m	0.02	0.05	0.10	0.25	0.40	0.50	0.60	0.75	0.90	0.95	0.98
24	3	3.10	3.46	3.83	4.67	5.57	6.32	7.27	9.56	15.36	22.10	34.39
	4	3.13	3.44	3.78	4.52	5.24	5.82	6.53	8.10	11.62	14.93	20.76
	5	3.12	3.41	3.73	4.44	5.09	5.59	6.20	7.45	10.09	12.23	15.75
	6	3.11	3.40	3.71	4.38	4.99	5.43	5.96	7.03	9.16	10.80	13.44
	7	3.11	3.39	3.70	4.34	4.92	5.33	5.79	6.72	8.53	9.97	12.10
	8	3.11	3.39	3.69	4.31	4.85	5.24	5.68	6.52	8.12	9.34	11.18
	9	3.11	3.40	3.69	4.29	4.81	5.17	5.57	6.36	7.80	8.90	10.37
	10	3.11	3.39	3.69	4.27	4.77	5.10	5.49	6.23	7.55	8.52	9.94
	11	3.10	3.40	3.68	4.26	4.74	5.07	5.43	6.11	7.35	8.26	9.49
	12	3.11	3.40	3.69	4.25	4.72	5.03	5.37	6.03	7.19	8.04	9.11
	13	3.13	3.41	3.69	4.25	4.70	4.99	5.32	5.95	7.07	7.86	8.84
	14	3.14	3.43	3.69	4.24	4.68	4.97	5.29	5.89	6.91	7.66	8.55
	15	3.16	3.43	3.71	4.25	4.66	4.95	5.26	5.82	6.79	7.47	8.30
	16	3.17	3.44	3.71	4.24	4.65	4.93	5.23	5.77	6.70	7.34	8.16
	17	3.19	3.44	3.72	4.24	4.64	4.91	5.20	5.74	6.60	7.25	7.99
	18	3.19	3.46	3.73	4.23	4.62	4.88	5.17	5.69	6.53	7.12	7.83
	19	3.22	3.48	3.74	4.23	4.62	4.87	5.14	5.64	6.44	7.00	7.72
	20	3.23	3.49	3.74	4.23	4.61	4.85	5.12	5.59	6.38	6.91	7.53
	21	3.25	3.50	3.75	4.23	4.60	4.83	5.10	5.54	6.30	6.81	7.47
	22	3.27	3.52	3.76	4.23	4.58	4.81	5.06	5.52	6.22	6.74	7.36
	23	3.28	3.53	3.77	4.24	4.58	4.80	5.03	5.48	6.15	6.63	7.24
	24	3.31	3.55	3.79	4.24	4.57	4.78	5.02	5.43	6.09	6.54	7.11
25	3	3.09	3.47	3.87	4.71	5.59	6.31	7.24	9.40	15.23	21.80	34.84
	4	3.14	3.45	3.80	4.56	5.27	5.82	6.50	8.00	11.51	14.84	20.21
	5	3.12	3.44	3.77	4.47	5.11	5.58	6.15	7.34	9.81	12.21	15.64
	6	3.12	3.44	3.74	4.43	5.01	5.44	5.94	6.97	9.00	10.75	13.41
	7	3.11	3.43	3.73	4.37	4.92	5.32	5.80	6.70	8.45	9.92	11.90
	8	3.13	3.42	3.72	4.34	4.87	5.24	5.67	6.48	8.03	9.28	11.01
	9	3.13	3.42	3.71	4.32	4.83	5.17	5.57	6.33	7.75	8.88	10.46
	10	3.13	3.43	3.72	4.30	4.79	5.12	5.50	6.22	7.51	8.51	9.95
	11	3.13	3.43	3.71	4.29	4.76	5.07	5.43	6.11	7.32	8.26	9.52
	12	3.15	3.43	3.71	4.26	4.72	5.05	5.38	6.03	7.16	8.04	9.15
	13	3.16	3.44	3.72	4.25	4.70	5.00	5.34	5.94	7.03	7.82	8.90
	14	3.17	3.45	3.72	4.25	4.69	4.97	5.30	5.88	6.90	7.64	8.68
	15	3.18	3.46	3.72	4.24	4.67	4.95	5.26	5.83	6.78	7.47	8.42
	16	3.18	3.47	3.73	4.24	4.66	4.93	5.22	5.76	6.69	7.35	8.23
	17	3.20	3.48	3.74	4.24	4.65	4.91	5.19	5.72	6.63	7.26	8.03
	18	3.23	3.48	3.75	4.23	4.63	4.90	5.17	5.68	6.54	7.14	7.88
	19	3.24	3.49	3.75	4.24	4.62	4.87	5.14	5.63	6.45	7.03	7.70
	20	3.25	3.50	3.75	4.24	4.61	4.85	5.11	5.58	6.38	6.91	7.57
	21	3.26	3.52	3.77	4.24	4.60	4.84	5.09	5.55	6.32	6.82	7.45
	22	3.27	3.54	3.78	4.24	4.58	4.82	5.07	5.51	6.25	6.75	7.35
	23	3.30	3.55	3.79	4.24	4.58	4.81	5.05	5.47	6.18	6.68	7.26
	24	3.31	3.57	3.80	4.23	4.58	4.79	5.02	5.45	6.13	6.58	7.19
	25	3.33	3.58	3.82	4.23	4.57	4.78	5.00	5.41	6.07	6.51	7.07